# Evaluation of Exclusive Bus Lanes in a Tri-modal Road Network Incorporating Carpooling Behavior

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**ABSTRACT**

This paper considers the evaluation of exclusive bus lanes (EBLs) in road network with three travel modes: bus, solo driving, and carpooling. A tri-modal transportation network equilibrium model is developed to analyze the effects of EBLs under three different policies: (i) no EBLs (called Policy 1); (ii) EBLs can only be used by bus (called Policy 2); and (iii) EBLs can be used by both bus and carpooling modes (called Policy 3). By taking into account both EBLs setting scheme and bus frequencies, a combinatorial optimization model is proposed to test the performance of the tri-modal transportation system. In a traffic corridor case with single O-D pair, numerical results show that travel demand levels will remarkably influence the total system costs under different policies. The effects of average carpooling occupancy and mode choice parameters on travelers' choice behavior are analyzed. Finally, a tri-modal network with nineteen links is used to illustrate that the system could be more efficient when different EBLs policies are adopted on different links.

**Keywords:** Exclusive bus lanes; tri-modal network; traffic equilibrium; carpooling behavior.

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## 1. Introduction

Traffic congestion is becoming more and more ubiquitous in metropolitan areas all over the world due to the ever-increasing car ownerships. Therefore, besides traditional public transportation, new travel modes and traffic management methods have been developed to alleviate urban congestion. Among them, exclusive bus lanes (EBLs) and carpooling are two common and effective methods. High occupancy vehicle (HOV) lane, which is usually used in freeways in United States to encourage travelers to choose carpooling instead of solo driving, has also been attempted in urban area of Shenzhen City in China recently ("Congested Chinese city to open carpool lane," 2016). Some cities share HOV and EBLs in one lane, such as Interstate 5 in Seattle, and route 116 in Lévis, Quebec ("High-occupancy vehicle lane," 2017). It is clear that re-assignment of road resources has become a major control measure/policy to increase the share of bus and carpooling modes so as to reduce traffic congestion.

EBLs is a topic that has been discussed by many researchers (Shalaby, 1999; Viegas and Lu, 2004; Eichlerand and Daganzo, 2006; Abdelghany et al., 2007; Arasan and Vedagiri, 2008, 2009, 2010; Vedagiri and Arasan, 2009; Li and Ju, 2009; Chen et al., 2010; Zhu, 2010; Basso et al., 2011; McDonnell and Zellner, 2011; Yao et al., 2012, 2015; Yu et al., 2015, Wu et al., 2017) in the past two decades. Most of them did not consider modal split and route choice qualitatively or only considered two modes of bus and car in their mathematical models. Besides, as another major way to improve efficiency of urban transportation, carpooling behavior is firstly considered as the way of fuel saving and operating cost decreasing (Ronald et al., 1974; Kocur and Hendrickson, 1983; Bento et al., 2013), and then as the way of relieving traffic congestion (Yang and Huang, 1999; Huang et al., 2000; Li et al., 2007; Konishi and Mun, 2010; Agatz et al., 2011, 2012; Burris et al., 2014; Xu et al., 2015a,b; Stiglic et al., 2016) and reducing vehicle emissions (Erdoğan et al., 2015). There are also some qualitative (Horowitz, 1976; Tischer and Dobson, 1979; Wang, 2011) and quantitative researches (Habib et al., 2011; Qian and Zhang, 2011, Neoh et al., 2017) incorporating bus transit, solo driving and carpooling modes. However, they have not considered EBLs setting and carpooling behavior simultaneously.

Relatively little attention has been paid to the evaluation of EBLs implementation policies in a road network with three travel modes: bus, solo driving, and carpooling. To fill up the gap mentioned above, this paper aims to address the following questions: (a) How to formulate a network equilibrium model incorporating both EBLs and carpooling behavior? (b) How to test and optimize the performance of transportation system under the tri-modal equilibrium model? (c) What is the best implementation policy of EBLs in a tri-modal road network? (d) What are the impacts of travel demand, choice behavior, and other factors on the share and cost of each travel mode?

Thus, in this paper, a tri-modal transportation network equilibrium model with EBLs is established. Two numerical cases are carried out by the proposed model to analyze the system performances under different policies: (i) no EBLs (called Policy 1); (ii) EBLs can only be used by bus (called Policy 2); and (iii) EBLs can be used by both bus and carpooling modes (called Policy 3).

The remainder of the paper is organized as follows. In Section 2, the tri-modal transportation network equilibrium model incorporating both carpooling behavior and EBLs setting schemes is established. Section 3 elaborates the evaluation methodology of EBLs based on the model proposed in Section 2. Based on the proposed model, in Section 4, how travel demand and choice preference affect optimal policies, share and cost of each travel mode are analyzed in a traffic corridor example. The sensitivities of average carpooling occupancy, and choice behavior parameters are also analyzed. Further, how the combinational usage of multiple policies in different links affects overall system efficiency is discussed in Section 4. Finally, general conclusions and future studies are summarized in Section 5.

## 2. Tri-modal transportation network equilibrium analysis incorporating carpooling behavior and EBLs setting schemes

### 2.1. Tri-modal transportation network introduction

When choosing between bus, solo driving and carpooling, travelers' final decision is correlated with general travel costs. The general travel costs of solo driving include travel time, fuel cost, congestion toll, parking toll and etc. Carpoolers share fuel cost, congestion toll and parking toll, but they have extra costs in coordinating travelling schedules, origins and destinations. The extra costs are referred to carpooling coordination cost. The general travel costs for bus passengers include walking cost, transferring cost, in-vehicle travel cost and in-vehicle congestion cost. For no EBLs case, all vehicles run simultaneously in road. When setting EBLs, bus can use EBLs independently or use EBLs with carpooling vehicles simultaneously.

Considering the road transportation network $(V, A)$, $V$ and $A$ denote node set and link set respectively. $rs \in RS \subset V \times V$ denotes origin-destination (O-D) pairs from origin $r$ to destination $s$. $G$ denotes bus line set, $g \in G$ is bus line number. Let $C_a$ denote capacity of single lane in link $a$, $n_a$ denote the number of lane in link $a$. For convenience, similar with Konishi and Mun (2010), $m$ carpoolers share one car. Let $Q_{rs}$, $q_{rs}^{c}$, $q_{rs}^{c1}$, $q_{rs}^{c2}$ and $q_{rs}^{b}$ denote travel demand of all travelers, automobile travelers, solo drivers, carpoolers and bus passengers respectively in O-D pairs $rs$, they satisfy:

$$Q_{rs} = q_{rs}^{c} + q_{rs}^{b} = q_{rs}^{c1} + q_{rs}^{c2} + q_{rs}^{b}, \qquad rs \in RS \tag{1}$$

Let $P_{rs}^{c}$ to be the route set of automobile, $f_p^{c1}$ and $f_p^{c2}$ denote the number of solo drivers and carpoolers on route $p$ ($p \in P_{rs}^{c}$) respectively. $y_a^{c1}$ and $y_a^{c2}$ are the number of solo drivers, and carpoolers in link $a$ ($a \in A$) respectively. They satisfy:

$$y_a^{ci} = \sum_{rs \in RS} \sum_{p \in P_{rs}^{c}} \delta_p^a f_p^{ci}, \qquad a \in A, i \in \{1,2\} \tag{2}$$

where $\delta_p^a = 1$ if route $p$ passes link $a$, otherwise $\delta_p^a = 0$.

For bus mode, $P_{rs}^{b}$ is the set of all bus travel routes from origin $r$ to destination $s$, note that different bus transfer schemes on the same path belong to different bus travel routes. Let $f_p^b$ to be the number of bus passengers by route $p$ ($p \in P_{rs}^{b}$), and $y_a^g$ denotes the number of passengers of bus line $g$ in link $a$. Their relations are as follows:

$$y_a^g = \sum_{rs \in RS} \sum_{p \in P_{rs}^{b}} \delta_p^{ga} f_p^b, \qquad a \in A, g \in G \tag{3}$$

$$y_a^b = \sum_{g\ in\ a} y_a^g\,, \qquad a \in A, g \in G \tag{4}$$

where $\delta_p^{ga} = 1$ if route $p$ passes link $a$ using bus line $g$, otherwise $\delta_p^{ga} = 0$, $y_a^b$ is the total number of bus passengers in link $a$.

$x_a^{c1}$, $x_a^{c2}$ represent vehicle flow of solo driving and carpooling modes in link $a$ respectively. It is assumed that only one person in a solo driving car and $m$ people in a carpooling vehicle, then $x_a^{c1} = y_a^{c1}$ and $x_a^{c2} = y_a^{c2}/m$. For bus vehicle flow in link $a$, $x_a^b$ satisfies

$$x_a^b = \sum_{g\in G} \delta_g^a F_g\,, \qquad a \in A \tag{5}$$

where $F_g$ is the frequency of bus line $g$, $\delta_g^a = 1$ only when bus line $g$ passes link $a$, otherwise $\delta_g^a = 0$.

### 2.2. Link travel time analysis of three modes with different EBLs' setting schemes

Three policies of road resource assignment are considered in this paper. Policy 1: No EBLs, all vehicles use road resource simultaneously; Policy 2: Setting EBLs and only bus can use them; Policy 3: Setting EBLs and both bus and carpooling vehicles can use them simultaneously.

With Policy 1, vehicles of three modes run simultaneously, the travel time for the two automobile modes are the same. Their link travel time on link $a$ can be written by US Bureau of Public Roads (BPR) function as follows:

$$t_a^{ci1} = t_{a0}^c\left(1 + \alpha^c\left(\frac{y_a^{c1} + \frac{1}{m}y_a^{c2} + Kx_a^b}{n_a C_a}\right)^{\beta^c}\right), \qquad a \in A, i \in \{1,2\} \tag{6}$$

where $K$ is the vehicle conversion factor for bus, $t_{a0}^c$ is the free flow travel time of automobile on link $a$, $\alpha^c$ and $\beta^c$ are BPR function parameters for automobile mode. Travel time for bus mode in link $a$ can be written as:

$$t_a^{b1} = t_{a0}^b\left(1 + \alpha^b\left(\frac{y_a^{c1} + \frac{1}{m}y_a^{c2} + Kx_a^b}{n_a C_a}\right)^{\beta^b}\right), \qquad a \in A \tag{7}$$

where $t_{a0}^{b}$ is the free flow travel time of bus on link $a$, $\alpha^{b}$ and $\beta^{b}$ are BPR function parameters for bus mode.

For Policy 2, the travel times for the two automobile modes are also the same and can be written as:

$$t_{a}^{ci2} = t_{a0}^{c}\left(1+\alpha^{c}\left(\frac{y_{a}^{c1}+\frac{1}{m}y_{a}^{c2}}{(n_{a}-1)C_{a}}\right)^{\beta^{c}}\right), \qquad a \in A \quad i \in \{1,2\} \tag{8}$$

The travel time for bus mode could be:

$$t_{a}^{b2} = t_{a0}^{b}\left(1+\alpha^{b}\left(\frac{Kx_{a}^{b}}{C_{a}}\right)^{\beta^{b}}\right), \qquad a \in A \tag{9}$$

For Policy 3, it is assumed that carpoolers always choose the fastest lane. In the vast majority of practical circumstances and all the cases presented in this paper, EBLs are less congested than common lanes. Under very high bus frequency or share of carpooling proportion, EBLs could be worse than common lanes. Then, some carpoolers would choose common lanes until the travel costs of two kinds of lanes reach to an equilibrium. Denote $y_{a1}^{c2}$ and $y_{a2}^{c2}$ to be the number of carpoolers on link $a$ use EBLs and common lanes respectively. They satisfy $y_{a1}^{c2}+y_{a2}^{c2}=y_{a}^{c2}$. The travel time for solo driving mode in link $a$ could be:

$$t_{a}^{c13} = t_{a0}^{c}\left(1+\alpha^{c}\left(\frac{y_{a}^{c1}+\frac{1}{m}y_{a2}^{c2}}{(n_{a}-1)C_{a}}\right)^{\beta^{c}}\right), \qquad a \in A, i \in \{1,2\} \tag{10}$$

The travel time for bus mode can be written as:

$$t_{a}^{b3} = t_{a0}^{b}\left(1+\alpha^{b}\left(\frac{\frac{1}{m}y_{a1}^{c2}+Kx_{a}^{b}}{C_{a}}\right)^{\beta^{b}}\right), \qquad a \in A \tag{11}$$

Using $t_{a1}^{23}$ and $t_{a2}^{23}$ to denote the travel cost of carpooling mode on EBLs and common lanes in policy 3 respectively. There are $t_{a2}^{23}=t_{a}^{c13}$ and

$$t_{a1}^{c23} = t_{a0}^{c}\left(1+\alpha^{c}\left(\frac{\frac{1}{m}y_{a1}^{c2}+Kx_{a}^{b}}{C_{a}}\right)^{\beta^{c}}\right). \qquad a \in A, i \in \{1,2\} \tag{12}$$

The travel cost for carpooling mode under policy 3 is

$$t_a^{c23} = \min(t_{a1}^{c23}, t_{a2}^{c23}). \tag{13}$$

In fact, if there are carpoolers on common lanes, there should be $t_a^{c23} = t_{a1}^{c23} = t_{a2}^{c23} = t_a^{c13}$. As the goal of EBLs is to facilitate the mobility of the privileged vehicles, when EBLs is no faster than common lanes, actions should be taken to make EBLs to function again (such as raising the admission standard for carpooling occupancy, or using Policy 2 directly).

Let $\lambda_a$ to be a decision variable of EBLs setting and usage policies, $\lambda_a = 0$ if Policy 1 is used, $\lambda_a = 1$ if Policy 2 is used and $\lambda_a = -1$ if Policy 3 is used. Then, the travel time of all modes for all policies can be uniformly written as:

$$t_a^{ci} = (1 - |\lambda_a|)t_a^{ci1} + |\lambda_a| \left( \frac{1 + \lambda_a}{2} t_a^{ci2} + \frac{1 - \lambda_a}{2} t_a^{ci3} \right), a \in A, i \in \{1,2\} \tag{14}$$

$$t_a^{b} = (1 - |\lambda_a|)t_a^{b1} + |\lambda_a| \left( \frac{1 + \lambda_a}{2} t_a^{b2} + \frac{1 - \lambda_a}{2} t_a^{b3} \right). \quad a \in A \tag{15}$$

### 2.3. The general route travel costs analysis for three modes

For solo driving mode, general route travel costs mainly include travel time, fuel fare, parking toll, etc. It can be written as:

$$e_p^{c1} = \sum_{a \in A} \delta_p^a \gamma^c t_a^{c1} + \Delta_p^c, \qquad p \in P_{rs}^c, rs \in RS \tag{16}$$

where $\gamma^c$ is the value of time for automobile traveler, $\Delta_p^c$ contains the remaining route costs, which is considered as a constant in this paper.

In addition to the route cost of driving alone, general route cost for carpooling also includes carpooling coordination cost. It can be written as follows:

$$e_p^{c2} = \sum_{a \in A} \delta_p^a \gamma^c t_a^{c2} + \frac{\Delta_p^c}{m} + \Delta_p^{c2}, \qquad p \in P_{rs}^c, rs \in RS \tag{17}$$

where $\Delta_p^{c2}$ is the carpooling coordination cost, which is also considered to be a constant in this paper.

Traveler's general route travel costs by bus mainly consist of walking time, transfer waiting time, travel time in vehicle, bus fare and vehicle comfort. In this paper, vehicle comfort is considered to be in-vehicle congestion effect (Lo et al., 2003). A generalized cost that contains in-vehicle congestion effect for bus line $g$ in link $a$ can be written as:

$$e_a^g = \gamma^b t_a^b \left(1 + \alpha \left(\frac{y_a^g}{BF_g}\right)^\beta\right), \qquad a \in A, \ g \in G \tag{18}$$

where $\gamma^b$ is value of time by bus, $B$ is capacity of a bus, $\alpha$, $\beta$ are parameters for congestion effect.

There are remaining costs for bus mode. Average waiting time at bus stop for bus line $g$ is $1/(2F_g)$. Let $d_p^b$ to be bus fare and $\Delta_p^b$ to be other additional costs for bus route $p$ (e.g., walking time at origin/destination and transfer points). Thus, the general travel cost for bus route $p$ can be written as:

$$e_p^b = \sum_{a \in A} \sum_{g \in G} \delta_p^{ga} e_a^g + \sum_{g \in G} \gamma^w \delta_p^g/(2F_g) + \gamma_d^b d_p^b + \Delta_p^b, p \in P_{rs}^b, rs \in RS \tag{19}$$

where $\gamma^w$, $\gamma_d^b$ are parameters that transfer average waiting time and bus fare to general travel cost respectively. $\delta_p^g$ is 0-1 variable, if bus route $p$ includes bus line $g$, then $\delta_p^g = 1$, otherwise $\delta_p^g = 0$. For simplify, $\Delta_p^b$ is considered to be a constant that has no relationship with passenger flow.

### 2.4. Multi-modal network equilibrium model incorporating carpool behavior

For the two automobile modes, travelers will choose routes which minimize their general travel costs. The equilibrium state can be written as:

$$\left(e_p^{ci} - e_{rs}^{ci}\right) f_p^{ci} = 0, \qquad p \in P_{rs}^c, rs \in RS, \qquad i \in \{1,2\} \tag{20}$$

$$f_p^{ci} \geq 0, \qquad p \in P_{rs}^c, rs \in RS \qquad i \in \{1,2\} \tag{21}$$

$$e_p^{ci} - e_{rs}^{ci} \geq 0, \qquad p \in P_{rs}^c, rs \in RS \qquad i \in \{1,2\} \tag{22}$$

$$\sum_{p \in P_{rs}^c} f_p^{ci} = q_{rs}^{ci}, \qquad rs \in RS \qquad i \in \{1,2\} \tag{23}$$

where $e_{rs}^{ci}$ is the minimal general route travel cost of the $i$ automobile mode from origin r to destination s. Automobile modes are solo driving for $i = 1$ and carpooling for $i = 2$.

The equilibrium of bus mode is similar with automobile modes:

$$\left(e_p^b - e_{rs}^b\right) f_p^b = 0, \qquad p \in P_{rs}^b, rs \in RS, \tag{24}$$

$$f_p^b \geq 0, \qquad p \in P_{rs}^b, rs \in RS, \tag{25}$$

$$e_p^b - e_{rs}^b \geq 0, \qquad p \in P_{rs}^b, rs \in RS, \tag{26}$$

$$\sum_{p \in P_{rs}^b} f_p^b = q_{rs}^b, \qquad rs \in RS, \tag{27}$$

where $e_{rs}^b$ the minimal general travel cost by bus from origin $r$ to destination s.

Additionally, under Policy 3, carpoolers' choice behavior between EBLs and common lanes are depicted in Equation (28) - (31).

$$(t_{ai}^{c23} - t_a^{c23})y_{ai}^{c2} = 0, \qquad a \in A \quad i \in \{1,2\} \tag{28}$$

$$y_{ai}^{c2} \geq 0, \qquad a \in A \quad i \in \{1,2\} \tag{29}$$

$$t_{ai}^{c23} - t_a^{c23} \geq 0, \qquad a \in A \quad i \in \{1,2\} \tag{30}$$

$$y_{a1}^{c2} + y_{a2}^{c2} = y_a^{c2}, \qquad a \in A \tag{31}$$

The Nested Logit Model is applied in the modal split process. The top level model is to choose between car (automobile modes) and bus, traffic demand for car and bus can be written as:

$$q_{rs}^c = Q_{rs} \frac{\exp(-\theta_1 e_{rs}^c)}{\exp(-\theta_1 e_{rs}^c) + \exp\left(-\theta_1 e_{rs}^b + \varphi_1\right)}, \qquad rs \in RS \tag{32}$$

$$q_{rs}^b = Q_{rs} - q_{rs}^c, \qquad rs \in RS \tag{33}$$

where $\theta_1$ determines the standard deviation of perceived error when choosing between car and bus, $\varphi_1$ adjusts the choice preference of the two modes (hereinafter referred to as bus preference parameter), $e_{rs}^c$ is the excepted travel cost of car modes (some times called "inclusive value") and can be written as:

$$e_{rs}^c = -\frac{1}{\theta_2} \ln(\exp(-\theta_2 e_{rs}^{c1}) + \exp(-\theta_2 e_{rs}^{c2} + \varphi_2)), \tag{34}$$

where $\theta_2$ is the dispersion of perceived error when choosing between solo driving and carpooling, $\varphi_2$ determines choice preference between the two automobile modes (hereinafter referred to as carpooling preference parameter). Traffic demand for solo driving and carpooling are split in the underlying level model because of their similarity, they can be calculated by

$$q_{rs}^{c1} = q_{rs}^c \frac{\exp(-\theta_2 e_{rs}^{c1})}{\exp(-\theta_2 e_{rs}^{c1}) + \exp(-\theta_2 e_{rs}^{c2} + \varphi_2)}, \qquad rs \in RS \tag{35}$$

$$q_{rs}^{c2} = q_{rs}^{c} - q_{rs}^{c1}, \qquad rs \in RS \tag{36}$$

Finally, the tri-modal transportation equilibrium model incorporating carpooling behavior is formulated by formulations (2)-(31). Similar to the case in Yao et al. (2012), it can be rewritten as a mathematical programming model as follows:

$$\begin{aligned}\min \sum_{i\in\{1,2\}} \sum_{rs\in RS} \sum_{p\in P_{rs}^{c}} (e_p^{ci} - e_{rs}^{ci}) f_p^{ci} + \sum_{rs\in RS} \sum_{p\in P_{rs}^{b}} (e_p^{b} - e_{rs}^{b}) f_p^{b} \\ + \sum_{i\in\{1,2\}} \sum_{a\in A} (t_{ai}^{c2} - t_a^{c2}) y_{ai}^{c2} \\ + \sum_{rs\in RS} \left( q_{rs}^{c} - \frac{Q_{rs}}{1 + \exp\left(\theta_1 \left(e_{rs}^{c} - e_{rs}^{b}\right) + \varphi_1\right)} \right)^2 \\ + \sum_{rs\in RS} \left( q_{rs}^{c1} - \frac{q_{rs}^{c}}{1 + \exp(\theta_2 (e_{rs}^{c1} - e_{rs}^{c2}) + \varphi_2)} \right)^2\end{aligned} \tag{37}$$

s.t. (2)-(19), (21)-(23), (25)-(27), (29)-(31), (33)-(34), (36).

### 2.5. Solution algorithm

As mentioned above, due to the mutual influence between various travel modes. It is difficult to solve this tri-modal transportation equilibrium problem by link-based algorithm because of the asymmetric feature. Thus, a route-based algorithm with method of successive average (MSA) is used to solve it. In each iteration, the proportions of different traffic modes are calculated by Nested Logit Model based on their current shortest general route cost. Next, traffic demand is assigned to the shortest route of each mode. Then, route flows are updated by MSA. The algorithm stops when satisfying the convergence criterion or reaching the maximum iteration number. Instead of the relative difference between two consecutive iterations, the relative difference between current state and equilibrium state is chosen to be the convergence criterion. Details of the algorithm are described as follow:

**Algorithm 1.** Algorithm for the multi-modal transportation network equilibrium model with carpooling behavior.

**Input:** Transportation network $(V, A)$; calibrators $n_a$, $C_a$, $\lambda_a$, $a \in A$; traffic demand $Q_{rs}$, $rs \in RS$ and split calibrators $\theta_1$, $\theta_2$, $\varphi_1$, $\varphi_2$; related parameters of bus: $G$, $F_g$, $B$, $K$, $P_{rs}^{b}$, $\delta_g^{a}$, $\delta_p^{ga}$, $t_{a0}^{b}$, $\alpha^b$, $\beta^b$, $\gamma^b$, $\gamma_d^b$, $\gamma^w$, $\alpha$, $\beta$, $d_p^b$, $\Delta_p^b$, $a \in A$, $g \in G$, $p \in P_{rs}^{b}$, $rs \in RS$; related parameters of automobiles: $P_{rs}^{c}$, $\delta_p^{a}$, $t_{a0}^{c}$, $\alpha^c$, $\beta^c$, $\gamma^c$, $\Delta_p^c$, $a \in A$, $p \in P_{rs}^{c}$, $rs \in RS$; related parameters of carpooling mode: $m$, $\Delta_{rs}^{c2}$; convergence precision $\varepsilon$ and maximum iteration number $N$.

**Output:** Bus related $q_{rs}^{b}$, $e_{rs}^{b}$, $f_p^{b}$, $p \in P_{rs}^{b}$, $rs \in RS$; solo driver related $q_{rs}^{c1}$, $e_{rs}^{c1}$, $f_p^{c1}$, $p \in P_{rs}^{c}$, $rs \in RS$; carpooling related $q_{rs}^{c2}$, $e_{rs}^{c2}$, $f_p^{c2}$, $p \in P_{rs}^{c}$, $rs \in RS$.

**Step 0**: Initialization.
  **for** $rs \in RS$:
    $q_{rs}^{c1} = q_{rs}^{c2} = q_{rs}^{b} = 0$;
    **for** $p \in P_{rs}^{c}$: $f_p^{c1} = f_p^{c2} = 0$;
    **for** $p \in P_{rs}^{b}$: $f_p^{b} = 0$;
  **for** $a \in A$:
    calculate $x_a^{b}$ using (5);
    initialize link flows to zeros;
    calculate link travel time for all three modes using (6) – (15);
  $n = 1$;

**Step 1**: Update general route cost.
  **for** $rs \in RS$:
    **for** $p \in P_{rs}^{c}$: calculate $e_p^{c1}$ using (16); calculate $e_p^{c2}$ using (17);
    **for** $p \in P_{rs}^{b}$: calculate $e_p^{b}$ using (19);
    $e_{rs}^{c1} = \min\{e_p^{c1} | p \in P_{rs}^{c}\}$;
    $e_{rs}^{c2} = \min\{e_p^{c2} | p \in P_{rs}^{c}\}$;
    $e_{rs}^{b} = \min\{e_p^{b} | p \in P_{rs}^{b}\}$;

**Step 2**: Update traffic demand for current iteration.
  **for** $rs \in RS$: calculate $\tilde{q}_{rs}^{b}$, $\tilde{q}_{rs}^{c1}$, $\tilde{q}_{rs}^{c2}$ using (32)-(36);

**Step 3**: Check convergence.
  **if** $n \geq N$ or

$$\frac{\sqrt{\sum_{rs\in RS}(q_{rs}^{b} - \tilde{q}_{rs}^{b})^2}}{\sum_{rs\in RS} Q_{rs}} + \frac{\sqrt{\sum_{rs\in RS}(q_{rs}^{c1} - \tilde{q}_{rs}^{c1})^2}}{\sum_{rs\in RS} q_{rs}^{c}} + \frac{\sum_{a\in A}\sum_{i\in\{1,2\}}(t_{ai}^{c23} - t_{a}^{c23})y_{ai}^{c2}}{\sum_{a\in A} t_a^{c23} y_a^{c23}} + \sum_{i\in\{1,2\}}\left(\frac{\sum_{rs\in RS}\sum_{p\in P_{rs}^{c}}\left(e_p^{ci} - e_{rs}^{ci}\right)f_p^{c}}{\sum_{rs\in RS} e_{rs}^{ci} q_{rs}^{ci}}\right) + \frac{\sum_{rs\in RS}\sum_{p\in P_{rs}^{b}}\left(e_p^{b} - e_{rs}^{b}\right)f_p^{b}}{\sum_{rs\in RS} e_{rs}^{b} q_{rs}^{b}} \leq \varepsilon:$$

    break;

**Step 4**: Perform an all-or-nothing assignment.
  **for** $rs \in RS$:
    $p_{rs}^{b} = \text{argmin}\{e_p^{b} | p \in P_{rs}^{b}\}$, $h^{b}(p_{rs}^{b}) = \tilde{q}_{rs}^{b}$; $h^{b}(p) = 0, p \in P_{rs}^{b} - p_{rs}^{b}$;
    $p_{rs}^{c1} = \text{argmin}\{e_p^{c1} | p \in P_{rs}^{c}\}$, $h^{c1}(p_{rs}^{c1}) = \tilde{q}_{rs}^{c1}$; $h^{c1}(p) = 0, p \in P_{rs}^{c} - p_{rs}^{c1}$;
    $p_{rs}^{c2} = \text{argmin}\{e_p^{c2} | p \in P_{rs}^{c}\}$, $h^{c2}(p_{rs}^{c2}) = \tilde{q}_{rs}^{c2}$; $h^{c2}(p) = 0, p \in P_{rs}^{c} - p_{rs}^{c2}$;

**Step 5**: Update route flow using MSA.
  **for** $rs \in RS$:

**for** $p \in P_{rs}^c$: $f_p^{c1} = \frac{n-1}{n} f_p^{c1} + \frac{1}{n} h^{c1}(p)$; $f_p^{c2} = \frac{n-1}{n} f_p^{c2} + \frac{1}{n} h^{c2}(p)$;
**for** $p \in P_{rs}^b$: $f_p^b = \frac{n-1}{n} f_p^b + \frac{1}{n} h^b(p)$;
update $q_{rs}^{c1}$, $q_{rs}^{c2}$, $q_{rs}^b$ and $q_{rs}^c$ using (23), (27), (36).

**Step 6**: Update link flow and travel time.
**for** $a \in A$:
update link flow of solo drivers and bus passengers by (2) and (3);
**if** $\lambda_a \neq -1$ or $n \leq 10$:

$$y_a^{c2} = \sum_{rs \in RS} \sum_{p \in P_{rs}^c} \delta_p^a f_p^{c2};$$

**else**:

$$j = \text{argmin}(t_{a1}^{c2}, t_{a2}^{c2}), y_{aj}^{c2} = \sum_{rs \in RS} \sum_{p \in P_{rs}^c} \delta_p^a f_p^{c2}, y_a^{c2} = y_{a1}^{c2} + y_{a2}^{c2};$$

update link travel time for all three modes using (6) – (15);
$n = n + 1$;
go to **Step 1**;

Note that the algorithm only starts to judge whether should assign carpooling vehicles to common lanes after 10 iterations. It is because that many common lanes have not been assigned with cars at the initial several iterations.

## 3. The evaluation of EBLs setting scheme based on tri-modal transportation equilibrium with carpooling behavior

In this paper, EBLs setting scheme incorporating carpooling behavior is evaluated by the total system travel cost, which is similar with Yao et al. (2012). The total system travel cost $E$ can be written as the sum of travelers' total general travel cost $E_1$ and bus operation cost $E_2$. In this paper, traffic modes include bus, solo driving and carpooling. Thus, travelers' total general travel cost $E_1$ can be written as:

$$E_1 = \sum_{rs \in RS} (e_{rs}^{c1} q_{rs}^{c1} + e_{rs}^{c2} q_{rs}^{c2} + e_{rs}^b q_{rs}^b).$$

The bus operation cost $E_2$ can be written as:

$$E_2 = \gamma_t^b \sum_{g \in G} F_g \sum_{a \in A} \delta_g^a t_a^b - \gamma_d^b \sum_{rs \in RS} \sum_{p \in P_{rs}^b} f_p^b d_p^b,$$

where $\gamma_t^b$ is the parameter which convert bus operating time to general travel cost.

If the optimal goal is to minimize total system travel cost, the decision variables are bus operation frequency of each bus line, setting scheme of EBLs and the adopted policy of EBLs. Then, the optimal decision model can be written as:

$$\min\ E = E_1 + E_2 \tag{38}$$

$$\text{s.t.}\ \ \lambda_a = 0, \pm 1 \qquad a \in A'$$

$$\lambda_a = 0 \qquad a \in A - A'$$

$$F_g^{\min} \leq F_g \leq F_g^{\max} \qquad g \in G$$

where $A'$ denotes the set of links passed by bus lines, $F_g^{\max}, F_g^{\min}$ denote respectively the upper and lower bound of bus operation frequency, and $q_{rs}^{c}$, $q_{rs}^{c1}$, $q_{rs}^{c2}$, $q_{rs}^{b}$, $e_{rs}^{c1}, e_{rs}^{c2}, e_{rs}^{b}, f_p^b, t_a^b$ can be obtained by the lower level model (37) with constraints (2)-(19), (21)-(23), (25)-(27), (29)-(31), (33)-(34), (36).

The optimal EBLs setting scheme with different implementation of policies can be obtained by the exhaustive method. For a large network, a genetic algorithm procedure similar to that of Yao et al. (2012) should be used for solving this model.

Note that the Equation (38) is only one of the optional optimal goals. The problem here is evaluated from a system/government's perspective, emphasizing on the practical benefits, therefore the total system cost $E$ is chosen to be minimized. Alternatively, if the problem is evaluated from the travelers' view, the optimization goal could be to minimize travelers' expected total system costs, as follows:

$$\min \sum_{rs\in RS} -\frac{1}{\theta_1} \ln(\exp(-\theta_1 e_{rs}^{b} + \varphi_1) + \exp(-\theta_1 e_{rs}^{c}))\, Q_{rs}.$$

## 4. Numerical Analysis and Evaluation

### 4.1. Traffic corridor evaluation with single O-D pair

In this section, how traffic demand and travelers' preference of different traffic modes affect optimal EBL setting policy, share of each mode and general travel cost is analyzed by a traffic corridor. Sensitivities of the main parameters are also analyzed.

4.1.1. Preliminary

Considering a traffic corridor with single O-D pair (e.g., from one city to another city) and two lanes as shown in Figure 1.

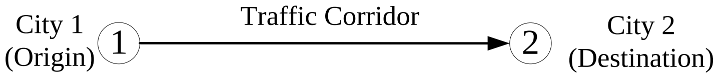


Fig 1 Traffic corridor with single O-D pair

Assuming total travel demand is $5000\ \eta$ persons per hour, free flow travel time by two automobile modes is 0.4 hour, free flow travel time by bus is 0.5 hour, capacity of each lane is 1200 passenger car unit, modal split parameters $\theta_1 = 3$, $\theta_2 = 4$, $\varphi_1 = \varphi_2 = 0$. For bus-related parameters, let $B = 40$, $K = 3$, $\alpha^b = 0.15$, $\beta^b = 4$, $\alpha = 0.1$, $\beta = 3$, $\gamma^b = 1$, $\gamma^w = 1.5$, $\gamma_d^b = 0.05$, $\Delta_p^b = 0.3$, bus fare for each transfer is 2 dollars, the highest and lowest frequency of bus are 60 and 5 vehicles per hour respectively. For parameters of two car modes, $\alpha^c = 0.15$, $\beta^c = 4$, $\gamma^c = 1$, $\Delta_p^c = 0.3$, $\Delta_p^{c2} = 0.3$, $m = 2$. Convergence precision $\varepsilon = 0.001$ and maximum iteration number $N = 1000$. Next, three different policies are compared under different demand levels and choice preference parameters.

4.1.2. Effect analysis of travel demand level

Based on the minimal total system cost model (33), the optimal frequency is obtained on each demand level. The total system costs (TSC) and the optimal frequencies (OF) with different travel demand levels are shown in Table 1. The performance of the algorithm described in Section 3 depends on demand level and bus frequency. At a low demand level (e.g. 0.2, 0.4) and bus frequency (e.g. 5-10), algorithm could converge within 10 iterations. In relatively high demand level and bus frequency case, the algorithm could stop in hundreds of iterations. All the results presented in Table 1 converge within 1000 steps. Note that in this example, travel time on EBLs is always lower than which on common lane for carpoolers in Policy 3, which means carpoolers will only use the EBLs.

Table 1 Total system costs and optimal frequencies with different travel demand levels in three policies

| $\eta$ | Policy 1 | | Policy 2 | | Policy 3 | |
|---|---|---|---|---|---|---|
| | TSC | OF | TSC | OF | TSC | OF |
| 0.2 | 790.1 | 15 | 793.0 | 16 | 790.7 | 15 |
| 0.4 | 1579.8 | 25 | 1647.3 | 35 | 1593.5 | 29 |
| 0.6 | 2428.0 | 33 | 2677.6 | 60 | 2464.4 | 43 |
| 0.8 | 3472.5 | 41 | 3834.5 | 60 | 3418.5 | 53 |
| 1 | 5042.2 | 47 | 5138.8 | 60 | 4497.7 | 58 |
| 1.2 | 8033.3 | 50 | 6729.2 | 60 | 5852.3 | 60 |
| 1.4 | 15272.4 | 42 | 8811.3 | 60 | 7859.0 | 60 |
| 1.6 | 31139.1 | 16 | 11623.1 | 60 | 11585.8 | 60 |
| 1.8 | 55501.5 | 5 | 15436.4 | 60 | 20260.5 | 60 |
| 2 | 91117.5 | 5 | 20557.5 | 60 | 39550.5 | 26 |

From Table 1, the following observations can be made:

- When travel demand is very low ($\eta = 0.2$), the total system costs for three policies are pretty close.
- Overall, with different travel demand level, best policy could be different. Best policy is Policy 1 when $\eta \leq 0.6$, Policy 3 when $0.8 \leq \eta \leq 1.6$ and Policy 2 when $\eta \geq 1.8$. The worst policy is Policy 2 when $0.2 \leq \eta \leq 1$ and Policy 1 when $\eta \geq 1.2$.

The share of each mode with different policies is illustrated in Figure 2.

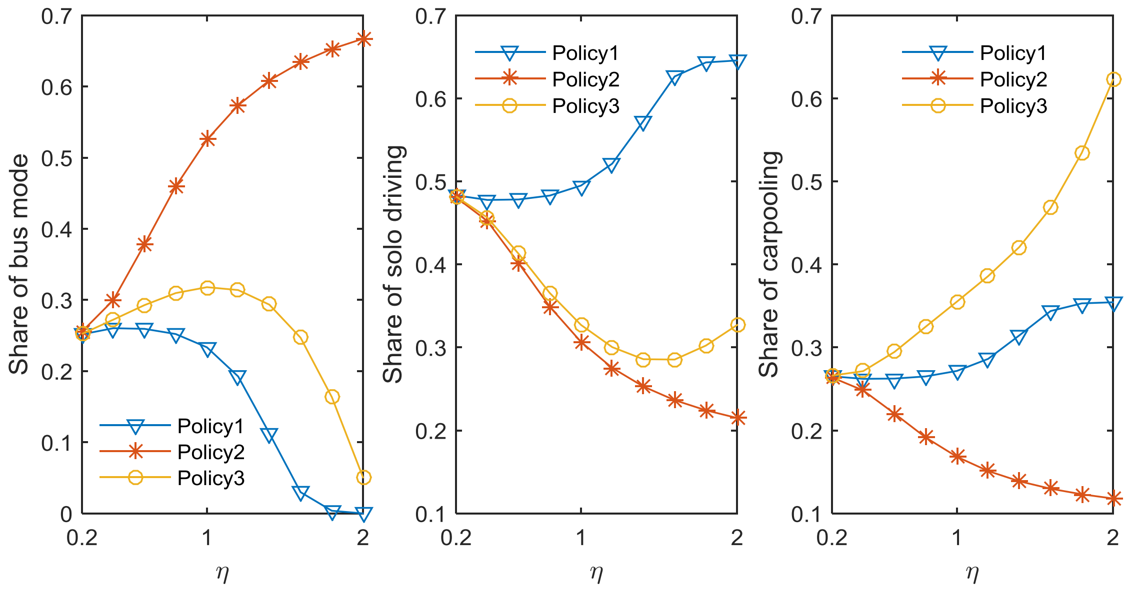


Fig. 2 Share of each mode in different policies

From Figure 2, the following observations can be made:

- With the increase of traffic demand, share of bus mode decreases noticeably in Policy 1 while grows significantly in Policy 2. The share in Policy 3 steadily increases, but begins to drop at high demand level (around $\eta > 1.3$). Share of bus mode is always highest in Policy 2 and lowest in Policy 1.
- With the increase of traffic demand, share of solo driving mode would increase in Policy 1 and decrease in policies 2. The share in Policy 3 first decreases, and begins to raise after a high demand level (around $\eta > 1.5$). Also, share of solo driving mode is always highest in Policy 1 and lowest in Policy 2.
- With the increase of traffic demand, share of carpooling mode increases in policies 1&3 and decreases in Policy 2. Share of carpooling mode is highest in Policy 3 and lowest in Policy 2.

Average general travel costs of three modes in different policies with different demand levels are shown in Figure 3.

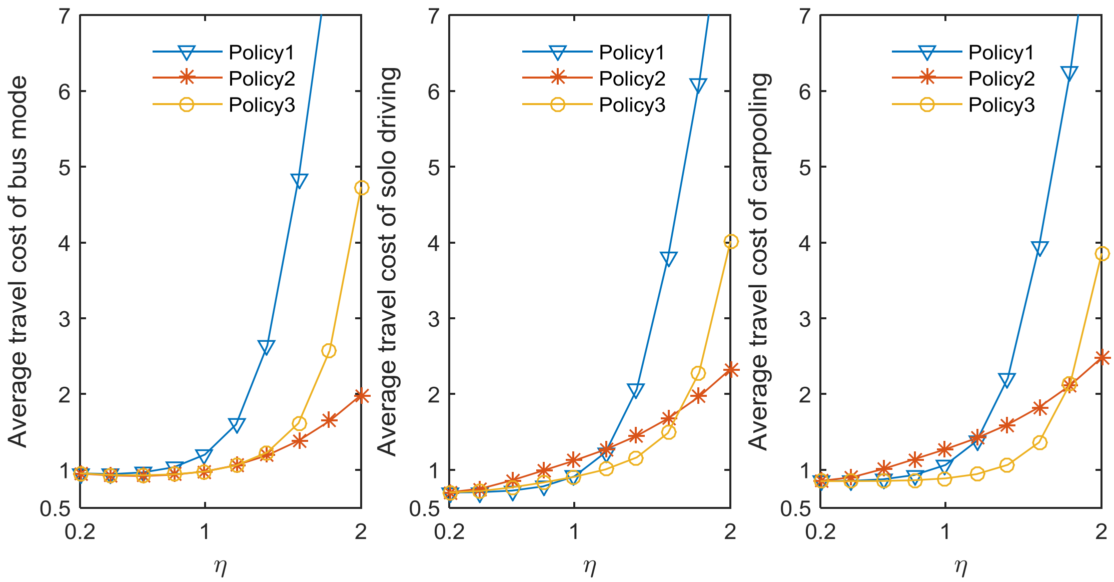


Fig. 3 Average general travel costs of three modes in different policies

It can be found from Figure 3 that at very low demand level (e.g. $\eta = 0.2$), average travel costs of three modes are very close in three policies. Besides, with the increase of traffic demand level, travel costs of all three modes under all three policies increase. However, the growth rate under different policies is quite different. For Policy 1, travel cost increase sharply (exponentially) after a relatively high demand level, and all three modes have the highest average general travel cost in Policy 1 when demand level is very high. Followed by Policy 3, whose travel cost also increase intensely in high demand level. Policy 2 has the highest travel cost of carpooling and solo driving at low demand level. The growth rate of travel cost in Policy 2, however, is the most moderate in high demand level. And all three traffic modes have the lowest average general travel cost in Policy 2 in high demand level.

The exponential growth in Policy 1 and Policy 3 can be deemed as the collapse of traffic system under such demand levels and policies. From this perspective, network under Policy 2 has the largest capacity, followed by Policy 3, and Policy 1 is the least suitable for high demand case. Note that this conclusion can also be gained from the total system costs in Table 1.

#### 4.1.3. Sensitivity analysis of choice preference parameters

In this section, how travelers' choice preferences (i.e. $\varphi_1$ and $\varphi_2$) affect flow equilibrium and general travel cost of each mode is analyzed. Because the purpose and significance of the proposed policies are to relieve travel congestion, analysis is conducted under congested situation when traffic demand level $\eta$ is $1.5$. Except $\varphi_1$ or $\varphi_2$, other parameters are the same with the previous section, the minimal total system cost is considered as a goal to obtain the optimal bus frequency in each policy.

In the Nested Logit Model, parameter $\varphi_1$ represents the choice preference of bus mode, bigger $\varphi_1$ indicates more travelers prefer to choose bus. The value of $\varphi_1$ is changed from -0.5 to 0.5, how $\varphi_1$ affects the share and average travel cost of each mode is shown in Figure 4.

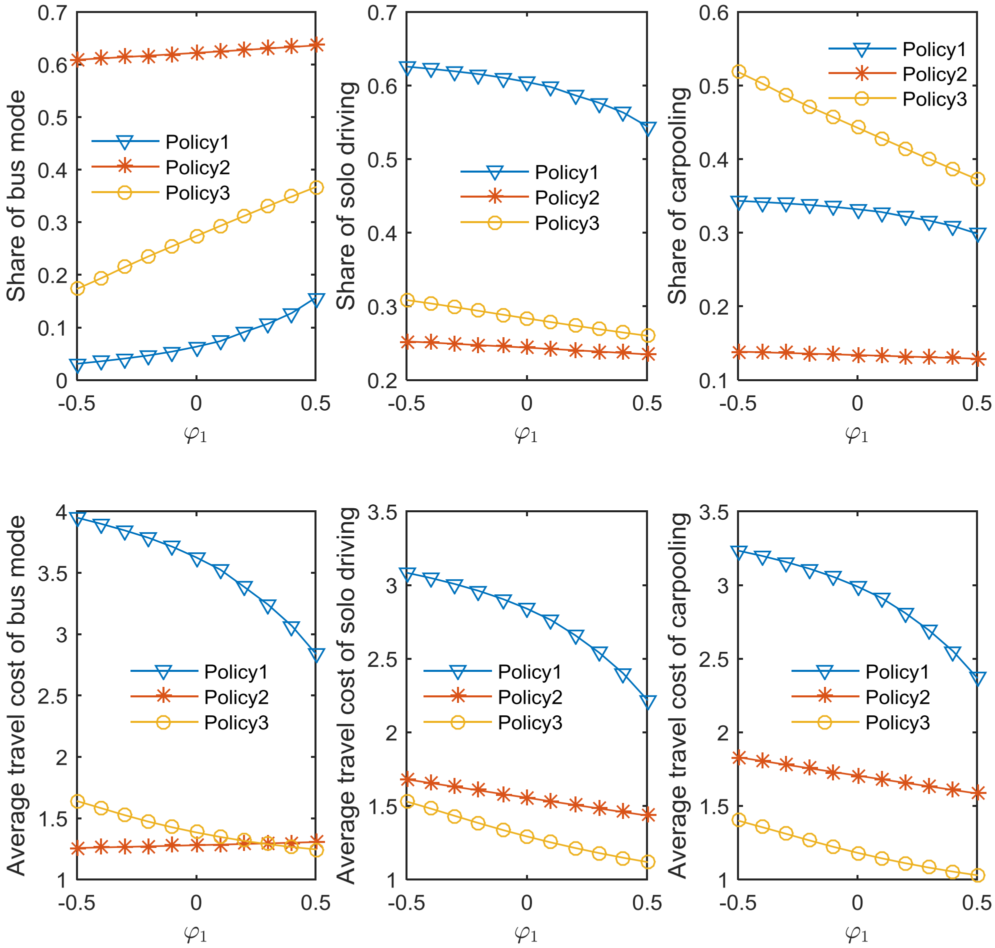


Fig. 4 Effect of bus preference on the share and average travel cost of each mode

From Figure 4, the following observations can be made:

- With the increase of $\varphi_1$, the share of bus increases in all three policies, whereas the share of the two car modes decreases in all three policies.
- Except bus mode of Policy 2, average travel costs of all three modes decline in all three policies, which could be explained as a consequence of the decrease in total vehicle number as more travelers choose bus to travel.
- Average travel cost of bus in Policy 2 exhibits a very slight rise with the increase of $\varphi_1$. Because bus is isolated from other ways of transport, reduction in car number does not alleviate congestion level of bus lane, the increase in the number of bus travelers slightly raises average bus travel cost.

Parameter $\varphi_2$ is the choice preference of carpooling mode, bigger $\varphi_2$ means more people prefer to use carpool. The value of $\varphi_2$ is changed from -0.5 to 0.5, how carpooling preference parameter $\varphi_2$ affects the share and average travel cost of each mode are shown in Figure 5.

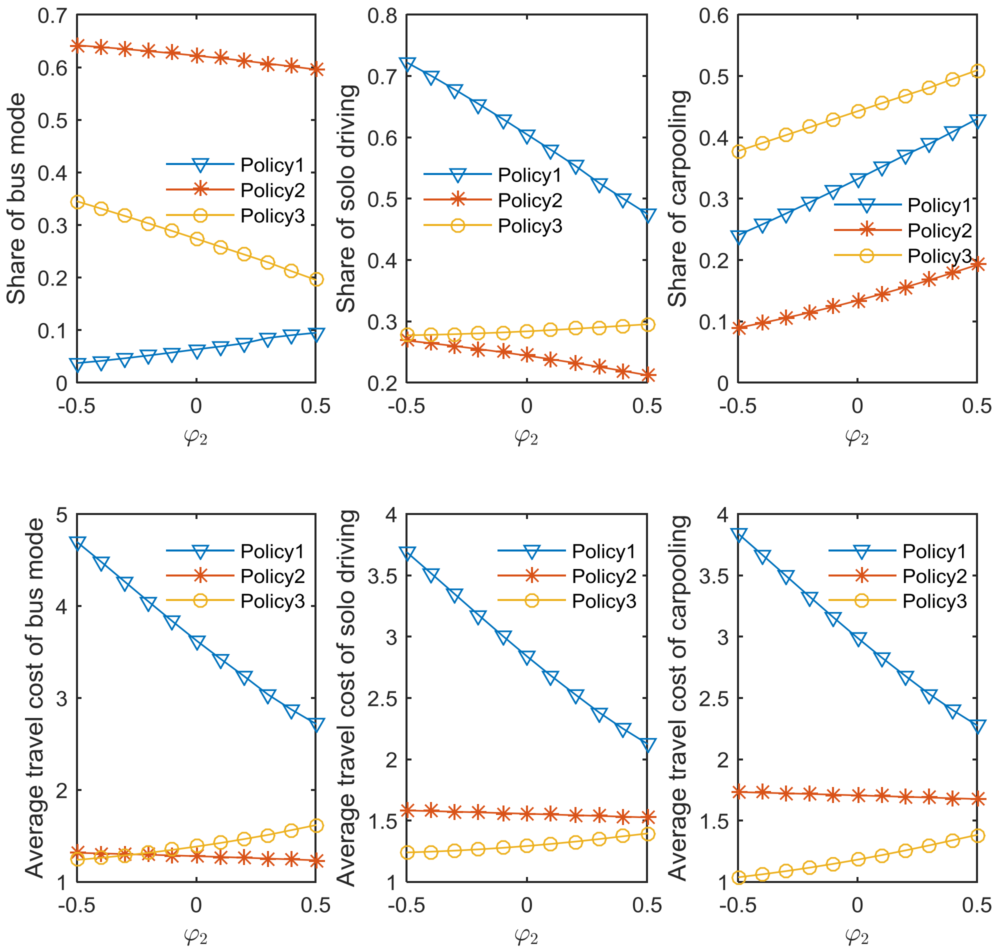


Fig. 5 Effect of carpooling preference on the share and average travel cost of each mode

From Figure 5, the following observations can be found:

- A bigger $\varphi_2$ results in a larger share of carpooling in all three policies.
- In Policy 1, average travel costs of all three modes decrease noticeably when increasing $\varphi_2$, which reflects that compared to solo driving, carpooling helps to reduce traffic congestion for all traffic modes under this policy.
- In Policy 2, when increasing $\varphi_2$, average travel costs of all three modes decrease slightly. It is because some bus passengers and solo drivers shift to carpooling mode.
- In Policy 3, average travel costs of bus and carpooling increase with the increase of $\varphi_2$, because the number of carpooling vehicles on EBLs increases. At the same time, the cost and the share of solo driving also increase when increasing $\varphi_2$, it can be accounted

by the increase in the cost of bus mode results in more travelers choose solo driving.

Overall, the analysis of $\varphi_1$ and $\varphi_2$ based on the proposed model above conforms to the results of the qualitative analysis and common sense. In addition, Figure 4 and Figure 5 show that Policy 2 gives steady guarantee to the average bus cost, which is almost unaffected.

Compare bus and carpool preference, it can be found that more preference on the bus mode generally decreases average travel costs of all modes in three policies. However, the increase in carpooling preference increases average travel costs of some modes at certain policy. How the two parameters affect the total system cost is depicted in Figure 6. It is clear that the total system costs under all three policies would decrease when more travelers use bus to travel, whereas total system cost may increase with Policy 3 when more travelers prefer to use carpooling mode.

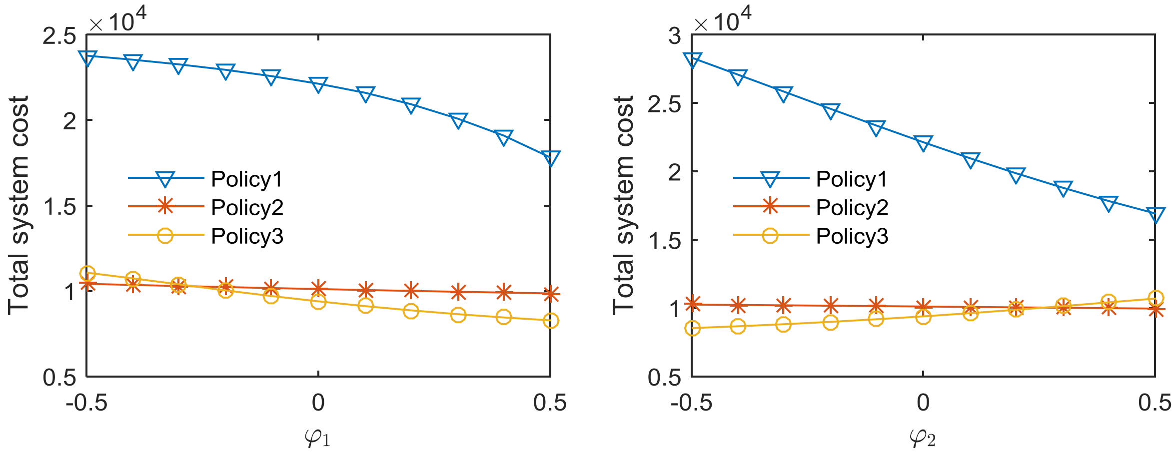


Fig.6 Effect of bus and carpooling preference on the total system cost

Under the same road resources, carpooling delivers more passengers than solo driving, but delivers less passengers than bus. It is understandable that increasing bus preference will increase system efficiency. For carpooling mode, carpoolers and solo drivers drive on the same lanes in Policy 1&2, when increasing carpooling preference, more travelers choose to carpool rather than to solo drive, road resources are saved and system efficiency is therefore enhanced. In Policy 3, bus and carpooling vehicle share the fixed road resources, when increasing carpooling preference, the efficiency of bus mode and carpooling mode will be reduced simultaneously, which leads the total system cost to increase.

Note that travelers have different sensitivities to the change of $\varphi_1$ and $\varphi_2$ under different $\theta_1$ and $\theta_2$, demand levels and other parameters, specific analysis should be made for any specific situation.

4.1.4. Sensitivity analysis of other parameters

To evaluate how the outputs of the proposed model change with different parameters, the sensitivities of average carpooling occupancy $m$, and choice dispersion $\theta_1$ and $\theta_2$ are analyzed. The analysis is conducted under a congested condition when travel demand level $\eta = 1.5$. When analyzing a certain parameter, other parameters are the same with the section 4.1.1.

When average carpooling occupancy $m$ increases, less cars are needed, and road will be less congested. It is interesting to study how these three policies respond to the change of $m$. The average carpooling occupancy is changed $m$ from 2 to 3 at interval 0.2, and the total system costs are calculated for three policies, the results are shown in Table 2. It can be found that the total system costs decrease with the increase of $m$ in all three policies, but with different sensitivities. Policy 1 is most sensitive to the change of $m$. Increasing carpooling occupancy also noticeably decreases the total system cost in Policy 3. Compared with the other two policies, the improvement brought by increasing carpooling occupancy is not very significant in Policy 2.

Table 2 Total system costs under different average carpool occupancies in three policies

| $m$ | 2 | 2.2 | 2.4 | 2.6 | 2.8 | 3 |
|---|---|---|---|---|---|---|
| Policy 1 | 22130.4 | 19631.1 | 17641.5 | 16115.8 | 14922.0 | 13976.0 |
| | 0.0% | -11.3% | -20.3% | -27.2% | -32.6% | -36.8% |
| Policy 2 | 10110.1 | 9998.1 | 9897.4 | 9805.8 | 9723.1 | 9648.0 |
| | 0.0% | -1.1% | -2.1% | -3.0% | -3.8% | -4.6% |
| Policy 3 | 9389.6 | 8589.8 | 8053.9 | 7676.0 | 7404.1 | 7193.0 |
| | 0.0% | -8.5% | -14.2% | -18.3% | -21.1% | -23.4% |

In the Nested Logit Model, $\theta_1$ and $\theta_2$ represent the dispersion of perceived error when choosing among different alternatives. The upper level $\theta_1$ is responsible for the split between bus and car, the lower level $\theta_2$ marks the dispersion of perceived error between solo driving and carpooling. Note the lower level $\theta_2$ should be no less than the upper level $\theta_1$, and the two levels are not independent. The effects of $\theta_1$ and $\theta_2$ can be more straightforward if the cost is assumed to be independent to the choice probability. Under this circumstance, bigger $\theta$ (smaller choice perceived error) will increase the probability in selecting the alternative with less cost. Unilaterally changing $\theta_1$ will not change the proportion of choice probability within the lower level (two car modes), but unilaterally changing $\theta_2$ will affect the choice probability in the upper level (bus and car). In the proposed model, however, the general travel cost depends on the choice probability, which makes the effect of $\theta_1$ and $\theta_2$ hard to be evaluated. Therefore, the effects of $\theta_1$ and $\theta_2$ in the proposed model are compared with the situation when travel cost is dependent to choice probability (hereinafter referred to as fixed-cost case).

Table 3 General travel costs of different modes under three policies when $\eta = 1.5$

| | $e^b$ | $e^c$ | $e^{c1}$ | $e^{c2}$ |
|---|---|---|---|---|
| Policy 1 | 3.626 | 2.730 | 2.840 | 2.990 |
| Policy 2 | 1.280 | 1.446 | 1.556 | 1.706 |
| Policy 3 | 1.384 | 1.058 | 1.293 | 1.181 |

For the traffic corridor example in section 4.1.1, the general travel costs of different modes under three policies when $\eta = 1.5$ are shown in Table 3, of which the perceived error dispersion parameters are $\theta_1 = 3$ and $\theta_2 = 4$ respectively. It can be seen that general travel cost of bus $e^b$ is bigger than the expected general cost of the two car modes $e^c$ in Policy 1&3, it is reversed in Policy 2. Assuming costs are fixed, then the share of the bus mode will decrease with the increase of $\theta_1$ in Policy 1&3, and will increase in Policy 2. This is depicted in Figure 7, where dash lines divide the share into three parts, each part represents the possibility of choosing a mode with different $\theta_1$ under the fixed-cost assumption. For the proposed variable-cost model, Figure 7 uses different colors to represent the share of each mode. It can be found that the general tendency of how the share of each mode changes with $\theta_1$ in the proposed model is consistent with the fixed-cost case. But, because of the interaction between cost and choice probability, the proposed model exhibits different sensitivity with regard to $\theta_1$ compared with the fixed-cost case.

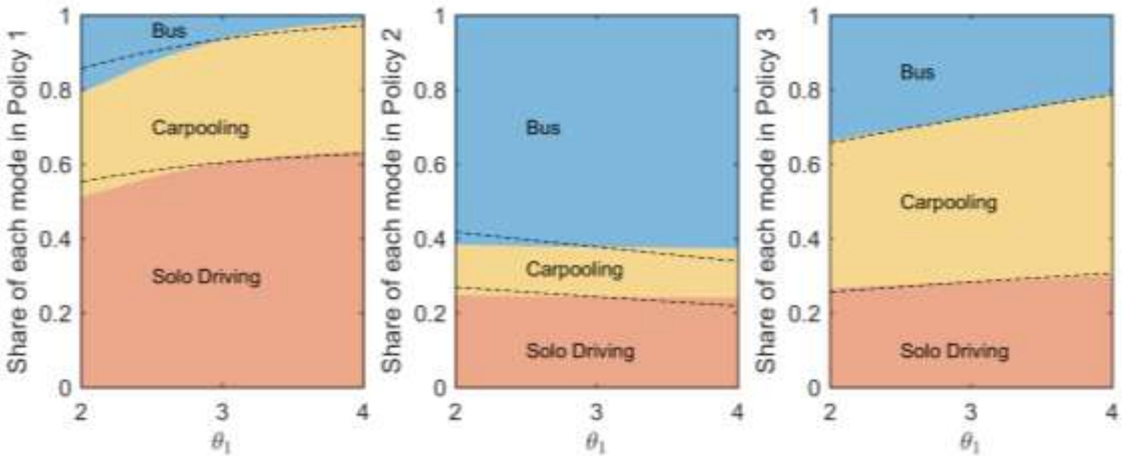


Fig. 7 Effect of $\theta_1$ on the share of each mode in three policies, dash lines represent the change in the share if general costs are fixed at the values in Table 3 ($\theta_1 = 3$)

Similarly, Figure 8 shows how the share of each mode changes with different $\theta_2$, and the case that costs are fixed is represented by dash lines. It can be found that the general effect of $\theta_2$ in the proposed model is similar to the fixed-cost case. However, the shares of some modes under certain policy have different change tendency compared with the situation if costs are assumed to be unchanged. For example, in Policy 1, the share of bus mode increases with the increase of $\theta_2$ in the fixed-cost case, but it is opposite in the proposed model. This is because solo driving is less costly than carpooling under the current situation of Policy 1 (shown in Table 3), raising $\theta_2$ increases the share of solo driving, which greatly increase the cost of bus, and the share of bus therefore decreases. The difference between the effect of $\theta_2$ in the proposed model and fixed-cost case can also be found in the change tendency of carpooling in Policy 3.

The overall effects of $\theta_1$ and $\theta_2$ in the proposed model are similar to the case when costs are fixed. A bigger $\theta$ generally increases the probability in choosing the alternative with less cost. But the interaction between choice probability and cost, which affects the sensitivity and even the change tendency of choice probability with regard to $\theta$, is also not negligible.

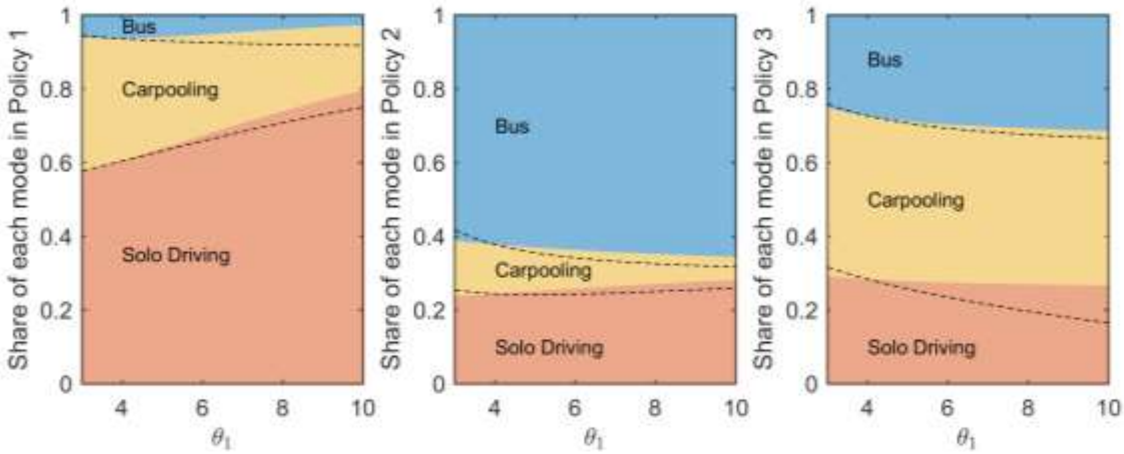


Fig. 8 Effect of $\theta_2$ on the share of each mode in three policies, dash lines represent the change in the share if general costs are fixed at the values in Table 3 ($\theta_2 = 4$)

### 4.2. Traffic network evaluation with multiple O-D pairs

The traffic corridor case in the last section illustrates that the best policy could depend on demand level which actually describes the road congestion level. In a general road network, congestion levels could be different for different links. Thus, applying the same policy for all links may not be the most efficient. As is shown Figure 9, the same network as Yao et al. (2012, 2015) is used to illustrate the effect of combinational EBLs setting policies. It has 13 nodes, 19 links and 6 bus lines, $v2$, $v4$, $v5$ and $v11$ are bus transfer nodes. The solid lines denote links for automobile, and the dotted lines denote bus lines. It is assumed each link has 3 lanes, and with the capacity of 400 pcu/h for each lane. The free flow travel time of the two automobile modes is 0.2 h on links 4 and 13, 0.3 h on link 18, and 0.1 h for other links. The free flow travel time of the bus mode is 1.2 times of which of the two automobile modes. The O-D demands are shown in Table 4. Other parameters are the same with the traffic corridor example.

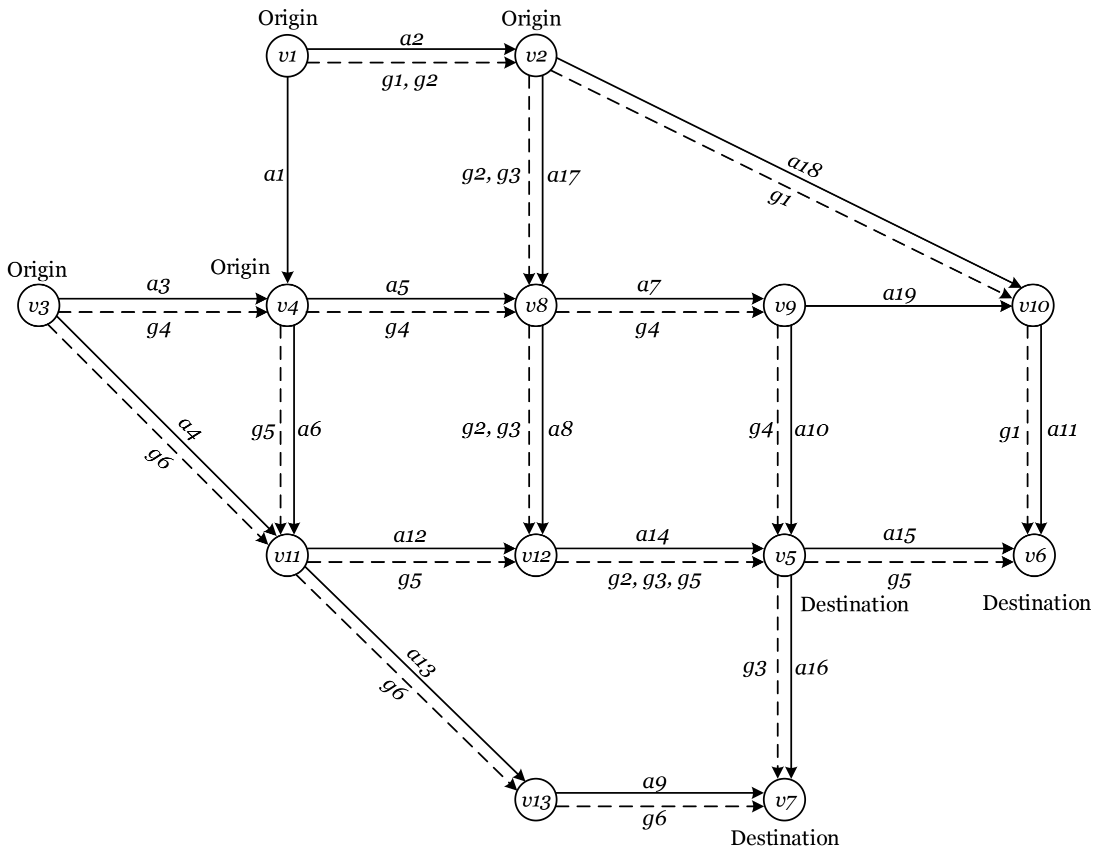


Fig. 9 A nineteen-link network

Table 4 O-D demands

| | Destinations | | |
|---|---|---|---|
| Origins | 5 | 6 | 7 |
| 1 | 900 | 600 | 300 |
| 2 | 1200 | 900 | 600 |
| 3 | 900 | 300 | 900 |
| 4 | 1200 | 600 | 600 |

Different combinational policies are compared in this network: (1) identical policy (only Policy 1, Policy 2 or Policy 3); (2) combinational usage of two policies (Policies 1&2, Policies 1&3 or Policies 2&3); (3) combinational usage of three policies (Policies 1&2&3). In this network, the algorithm normally reaches to the convergence of 0.01 after 1000 iterations, but it is hard to converge to 0.001 within 1000 iteration. Optimal bus frequencies and EBLs setting scheme for each situation are obtained by genetic algorithm. Optimal solutions for all the cases are shown in Table 5. It is clear that the smallest system total cost is obtained by combinational usage of three policies.

Table 5 Total system costs and optimal bus schemes with different combinational policies

| Policies | TSC | OF | Optimal EBLs Setting Scheme for link 2 to link 18 |
|---|---|---|---|
| Policy 1 | 7397.8 | 24, 14, 27, 17, 27, 9 | 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 |
| Policy 2 | 7269.3 | 34, 20, 39, 29, 41, 25 | 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2 |
| Policy 3 | 7204.4 | 22, 13, 26, 15, 26, 11 | 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3 |
| Policies 1&2 | 7110.4 | 24, 18, 36, 5, 45, 28 | 2, 1, 2, 1, 2, 1, 2, 1, 1, 1, 2, 1, 2, 1, 2, 2, 1 |
| Policies 1&3 | 7204.2 | 22, 12, 26, 15, 25, 11 | 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 1, 1, 3, 3 |
| Policies 2&3 | 7086.6 | 24, 19, 36, 5, 43, 27 | 2, 3, 3, 3, 3, 3, 2, 3, 3, 3, 2, 3, 2, 2, 2, 2, 3 |
| Policies 1&2&3 | 7086.5 | 24, 18, 36, 5, 44, 26 | 2, 3, 3, 3, 3, 3, 2, 3, 3, 3, 2, 3, 2, 1, 2, 2, 3 |

To evaluate the effect of different demand levels to the bus setting scheme, the O-D demands shown in Table 4 are multiplied by a parameter $\eta$. Let $\eta$ varies from 0.8 to 1.8 at the interval of 0.2, optimal total system costs and optimal bus schemes are shown in Table 6. With the increase of demand level, the optimal frequency of bus line 1, 3 and 5 steadily increase to the maximum frequency, serving as the primary bus lines. Meanwhile, total system cost increases with the increase of travel demand. Generally, it can be found that policy 2 is more widely used in high demand cases. Particularly, the best policy for link 2, 8, 14 and 17 is always policy 2 under all the demand levels. The common character for the four links is that there are multiple bus lines on these links. It suggests that policy 2 is more suitable for bus-intensive links.

For some links with policy 1 and policy 3 in Table 5 and Table 6, we found that the flows of bus passengers (not shown in the tables) are near to zeros, and the EBLs become “exclusive carpooling lanes”. On the on hand, it reflects that these bus lines are less efficient compared with other bus lines under these conditions. On the other hand, this is a shortcoming of the model. As it is assumed that only the bus route with smallest cost will be chosen by passengers. In reality, however, possibilities exist for choosing alternative bus routes with higher cost. It should be improved by applying more advanced transit equilibrium model in the future.

Further, for all the trials shown in Table 6, the number of travelers on each link (exclude link 1 and link 19, because there is no bus line on those links) are collected and grouped by different EBLs policies. Figure 10 shows the distribution of the number of travelers on a link under different policies by boxplots. On each box, the central mark indicates the median, and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. The two whiskers indicate the lowest datum still within 1.5 IQR (interquartile range) of the lower quartile and the highest datum still within 1.5 IQR of the upper quartile, respectively. The red dot represents the mean, and the outliers are plotted individually using the '+' symbol. It can be found that policy 2 is more often applied to links with high volume, and policy 1 is more suitable for low volume links; the observation is similar to the traffic corridor case.

Table 6 Total system costs and optimal bus schemes
at different demand levels

| $\eta$ | TSC | OF | Optimal EBLs Setting Scheme for link 2 to link 18 |
|---|---|---|---|
| 0.8 | 5509.7 | 21, 17, 32, 5, 36, 22 | 2, 1, 3, 3, 3, 3, 2, 3, 1, 3, 3, 3, 2, 3, 2, 2, 3 |
| 1.0 | 7086.5 | 24, 18, 36, 5, 44, 26 | 2, 3, 3, 3, 3, 3, 2, 3, 3, 3, 2, 3, 2, 1, 2, 2, 3 |
| 1.2 | 8809.5 | 25, 19, 39, 5, 52, 30 | 2, 3, 3, 3, 2, 1, 2, 3, 1, 3, 2, 3, 2, 1, 2, 2, 3 |
| 1.4 | 10651.5 | 56, 5, 53, 42, 55, 5 | 2, 1, 3, 2, 2, 2, 2, 3, 2, 2, 2, 3, 2, 1, 2, 2, 2 |
| 1.6 | 12439.7 | 60, 5, 57, 45, 60, 5 | 2, 1, 3, 2, 2, 2, 2, 3, 2, 2, 2, 3, 2, 2, 3, 2, 2 |
| 1.8 | 14263.3 | 60, 5, 60, 56, 60, 5 | 2, 1, 3, 2, 2, 2, 2, 3, 2, 2, 2, 3, 2, 2, 2, 2, 2 |

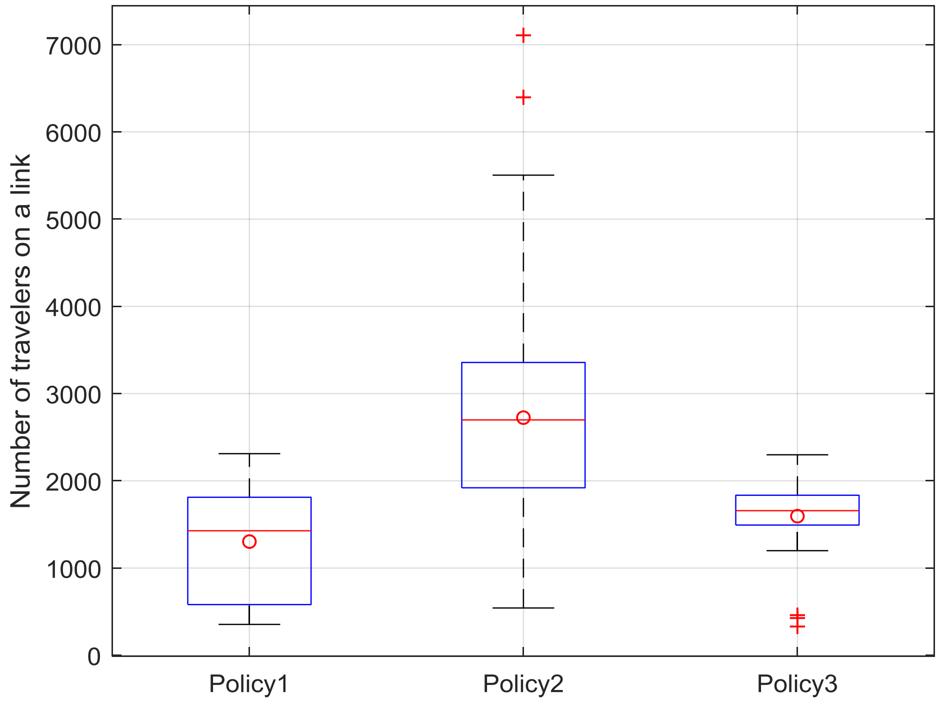


Fig. 10 The number of travelers on different links grouped by different policies

At last, it is important to clarify that the choose probabilities under different demand levels largely rely on the general travel costs and the increasing rates of general travel cost with regard to the volume of travelers. These two parts are the big background of the parameter setting, and the model output could be different if the background is changed. Specific analysis should be made for any specific situation.

## 5 Conclusions

In this paper, a multi-modal network equilibrium model, which incorporates bus, solo driving and carpooling, was established to analyze the effects of EBLs under different policies: (i) no EBLs (called Policy 1); (ii) EBLs can only be used by bus (called Policy 2); and (iii) EBLs can be used by both bus and carpooling (called Policy 3). The three policies were compared under their best bus frequencies, which are obtained by optimizing the total system cost.

Main conclusions in the two numerical examples are summarized into the following four aspects. Firstly, in the traffic corridor case with single O-D pair, the proposed model shows that the best policy depends on travel demand level. With the increase of demand level, the best policy would shift from Policy 1 to Policy 3, and finally to Policy 2 (network under Policy 2 has the largest capacity, followed by Policy 3, Policy 1 is least suitable for high demand situation). In addition, among the three policies, Policy 1 has the highest share for solo driving, Policy 2 has the highest share of bus, and Policy 3 has the highest share of carpooling. Secondly, the analysis of choice preference indicates that more travelers by bus would diminish the total system costs in all three policies, whereas the total system cost of policies 3 may increase when more travelers prefer to choose carpooling. Thirdly, higher average carpooling occupancy means a higher system efficiency for all three policies, and Policy 1 is most sensitive to the change of carpooling occupancy. Besides, the interaction between travel cost and choice probability affects the sensitivity of $\theta_1$ and $\theta_2$ in the proposed model. Finally, a more general tri-modal network shows that the smallest total system cost could be obtained by combinational usage of three policies in different kinds of links.

Further studies include applying more advanced transit equilibrium model, developing more efficient algorithm to improve the applicability of the proposed model on large-scale networks, such as algorithms with route generation process (e.g., Bovy and Fiorenzo-Catalano, 2007; Bekhor et al., 2008; Bovy, 2009). This model could also be extended to consider variable passenger numbers in carpooling vehicle, heterogeneity in the carpooling coordination cost (Konishi and Mun, 2010) and different time values or risk aversions for different travelers (Lo, Luo, and Siu 2006; Carrion and Levinson 2012). Further, advanced models could be developed to optimize the layout of bus lines based on the proposed model (e.g., Yu et al., 2015).

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