TRAFFIC ASSIGNMENT PARADOX INCORPORATING CONGESTION AND STOCHASTIC PERCEIVED ERROR SIMULTANEOUSLY

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ABSTRACT

This paper analyses the effects of congestion and stochastic perceived error in stochastic traffic assignment paradox, by the measure of both actual and perceived travel cost. Two different circumstances are studied: improving an existing link and adding a new link. It is found that different congestion cost functions and perceived error levels will significantly affect the road condition and the demand level under which paradox happens. Moreover, how the interaction between traffic demands of different O-D pairs affects the occurrence of traffic paradox is illustrated by a two O-D pairs' network. Besides, a counter-intuitive phenomenon when less stochastic perceived error yet increases the average travel cost (information paradox) is also discussed. The results of this paper help to understand the interactional impact of congestion and stochastic perceived error, and give some new insights to traffic paradox.

Keywords: traffic paradox; stochastic user equilibrium; congestion effect; stochastic perceived error

1. Introduction

Traffic researchers have long been puzzled with the phenomenon when adding or improving a

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link in a road network may unexpectedly degrade the network's performance, namely the traffic paradox. One of the most famous traffic paradoxes is the Braess' paradox (Braess, 1968; Braess et al., 2005), which results from the discrepancy between the user equilibrium (UE) and the system optimal (SO) conditions. Many researches have been conducted to dig deeper understanding of Braess' paradox (Frank, 1981; Steinberg and Zangwill, 1983; Dafermos and Nagurney, 1984; Pas and Principio, 1997; Zverovich and Avineri, 2015). Besides, the Braess' paradox has also been inspected under different conditions, such as Braess' paradox under elastic demand assignment (Hallefjord et al., 1994; Yang, 1997), dynamic/time-dependent traffic assignment (Arnott et al., 1993; Nagurney and Parkes, 2007), combined distribution and assignment (Yang and Chen, 2009), boundedly rational user equilibrium (Di et al., 2014).

There are other traffic paradoxes that focus on different types of problem, causes of them are various, including stochastic assignment paradox (Sheffi and Daganzo, 1978; Sheffi, 1985; Prashker and Bekhor, 2000; Yao and Chen, 2014, Zhao et al., 2014), modal split paradox (Fisk, 1979), capacity paradox (Yang et al., 1998), emission paradox (Nagurney, 2000), transit assignment paradox (Szeto and Jiang, 2014; Jiang and Szeto, 2016), maritime transportation (Balland et al., 2015), reliability paradox (Yin and Ieda, 2002; Szeto, 2011), exclusive bus lanes' setting paradox (Yao et al., 2015), and noise paradox (Wang and Szeto, 2017).

The stochastic assignment paradox caused by the stochastic perceived error of travellers was firstly proposed by Sheffi and Daganzo (1978), it can happen in an uncongested network. Prashker and Bekhor (2000) compared the stochastic assignment paradox with the UE and SO solutions using non-linear cost function, they concluded that the paradox only occurs in a certain range of demand volume. The feature of stochastic assignment paradox is further analysed by Yao and Chen (2014) and Zhao et al. (2014) in uncongested cases. In addition, paradoxes that involve the stochastic perceived error include modal split paradox (Fisk, 1979), combined distribution and assignment paradox (Yang and Chen, 2009), exclusive bus lanes' setting paradox (Yao et al., 2015), etc.

Although there have been many studies about the stochastic assignment paradox, little literature has discussed traffic assignment paradox incorporating congestion and stochastic perceived error simultaneously in details. This paper tries to bridge this gap, and inspects some phenomena that occur only when both factors are present. For example, the paradox measured by perceived travel cost in congested network and the "information paradox". The information paradox in this paper refers to when less perceived error (more information) yet increases average travel cost. This phenomenon has long been noticed (Boyce, 1988), and has become increasingly valuable in the age when Advanced Traveller Information Systems (ATIS) prevails in major cities. Previous studies mainly focus on the potential actions (departure time, change route, etc.) taken by travellers and the corresponding consequences after receiving the information. The methods applied include Vickrey equilibrium (Arnott et al., 1991), user equilibrium (Lindsey et al., 2014) and simulation (Rapoport et al., 2014). Our study, instead, using the perceived error to mark the amount of information gained by travellers and to analyse the condition for information paradox to happen.

We try to answer the following questions: (1) How does the congestion affect the stochastic assignment paradox? (2) Does the paradox corresponding to the average perceived travel cost exist? (3) How does the interaction between traffic demands from different O-D pairs affect the traffic paradox? (4) How does the change of stochastic perceived error affect the total travel cost in a congested network? The answers of these questions are given by analysing the paradox in a two-link network, the Braess' network and a two O-D pairs' network.

The reminder contents are organized as follows. Section 2 gives a brief review of the SUE model and its paradox. Section 3 studies the stochastic assignment paradox under the criterion of actual travel time. Section 4 further analyses the paradox by the measure of perceived travel time. A multiple O-D pairs' case is studied in Section 5. Section 6 discusses the phenomenon when less perceived error increases average travel cost, i.e. the information paradox. Finally, main conclusions are summarised in Section 7.

2. Some definitions of SUE and its paradox

Daganzo and Sheffi (1977) proposed the concept of SUE; the stochastic equilibrium conditions are reached when

$$f_k^i = P_k^i Q^i; \quad \forall k \in K^i \quad i \in I, \tag{1}$$

where f_k^i is the flow of route k in O-D pair i, P_k^i is the probability of choosing route k in the O-D pair i, Q^i is travel demand of O-D pair i, I and K^i are O-D pair set and route set of O-D pair i respectively. The route choice probability is based on discrete choice model and can be obtained by

$$P_k^i = \Pr\left(C_k^i \le C_l^i, \forall l \in K^i \text{and } k \ne l \right), \tag{2}$$

where C_k^i is the perceived travel cost of route k in O-D pair i. In random utility theory, it is conventionally assumed that $C_k^i = c_k^i + \xi_k^i$, where c_k^i and ξ_k^i are the actual travel cost and the perceived error of route k in O-D pair i. When assuming perceived error satisfies independent identical Gumbel distribution (Multinomial Logit Model), then

$$P_k^i = \frac{e^{-\theta c_k^i}}{\sum_{l \in K^i} e^{-\theta c_l^i}}; \quad \forall k \in K^i \quad i \in I, \tag{3}$$

where θ is a scale parameter relates to the variance of the stochastic perceived error. To obtain the SUE solution, regular network constrains also need to be hold:

$$C_k^i = \sum_{a \in A} T_a \delta_{ak}^i , \forall k \in K^i , i \in I \tag{4}$$

$$x_a = \sum_{i \in I} \sum_{k \in K^i} f_k^i \delta_{ak}^i ; \quad \forall a \in A \tag{5}$$

$$t_a = t(x_a), \tag{6}$$

where T_a is the perceived travel cost of link a, x_a represents the link flow, f_k^i is the flow of route k in O-D pair i, δ_{ak}^i=1 if route k of O-D i pass through link a, otherwise δ_{ak}^i=0, A is link set, t_a is the actual travel cost of link a. Because the mean of perceived error is assumed to be zero, we have $E(T_a) = t_a$.

At the state of user equilibrium, no travellers can improve his or her travel cost by unilaterally changing routes. In stochastic user equilibrium, however, every traveller minimises his or her perceived travel cost. Since the perceived travel cost of a route is a random variable, we concern about the average/expected perceived travel cost of an O-D pair. The average perceived travel cost of O-D i can be expressed as follows:

$$\widetilde{C^i} = E\left[\min_{k\in K^i}\{C_k^i\}\right]. \tag{7}$$

$\widetilde{C^i}$ captures the average perceived travel cost of a randomly selected traveller between O-D pair i. As discussed by Sheffi and Daganzo (1978), in a fixed-cost network, the partial derivative of the average perceived travel cost of an O-D pair with respect to the actual travel cost of a route equals the probability to choose that route, that is:

$$\frac{\partial\widetilde{C^i}}{\partial c_k^i} = P_k^i;\;\; \forall k \in K^i. \tag{8}$$

Using this property, we can derive the expression of average perceived travel cost. The average perceived travel cost of O-D pair i for Logit model is:

$$\widetilde{C_L^i} = -\frac{1}{\theta}\ln\sum_k e^{-\theta c_k^i}. \tag{9}$$

Sheffi and Daganzo (1978) have demonstrated that the stochastic paradox will never happen in the non-congested network when measured by average perceived travel cost. We will continue the discussion in the congested network. In all the examples presented in this paper, we use the Multinomial Logit route choice model. Because of the complexity of the SUE problem, the equilibrium state is obtained by the method of successive averages (MSA) algorithm. The algorithm stops when the relative distance between the last two iterations satisfies

$$\frac{\sqrt{\sum(x_a^{n+1} - x_a^n)^2}}{\sum x_a^n} \leq 10^{-5} \tag{10}$$

or maximum iteration number (1000) is reached. Since only small networks are applied in this paper, the algorithm converges very well after an acceptable number of iterations.

Like congestion effect, stochastic perception error is also one of the most essential factors affect the traffic distribution. It has been increasingly taken into account in the study of the traffic network (Jansuwan and Chen, 2015). Note that a larger θ value means a smaller variance in travellers' stochastic perceived error. When travellers get more information, the stochastic perceived error will become smaller (θ value will become lager). There is a counter-intuitive phenomenon that a smaller perceived error (more information) may lead to a higher average actual travel cost. We refer this phenomenon as information paradox, and it is evaluated by analysing the change of total actual travel cost with different levels of perceived error.

3. The paradox measured by actual travel cost

In this section, we use average actual travel cost as the criterion to measure the network's performance. Under this criterion, we investigate the feature of stochastic assignment paradox when congestion effect is introduced under two different circumstance: improving an existing link and adding a new link.

3.1 Improving a link

Similar to Sheffi and Daganzo (1978) and Yao and Chen (2014), the simple two-link network shown in Figure 1 is used in this section.

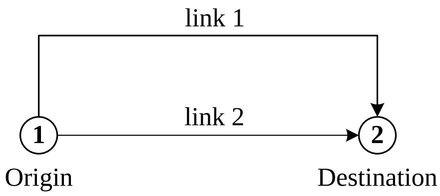


Figure 1 Two-link network

To evaluate the impact of congestion effect on the stochastic assignment paradox, we compare the paradox area of the two-link network under fixed cost $t_a = t_{0a}$, linear cost function $t_a = t_{0a}\left(1 + \frac{x_a}{1000}\right)$ and BPR function $t_a = t_{0a}\left(1 + 0.15\left(\frac{x_a}{1000}\right)^4\right)$, where t_a and t_{0a} are the actual and free flow travel cost of link a respectively; x_a is the flow volume of link a. The average travel cost of the two-link network can be expressed as follows:

$$\bar{C} = t_1 p_1 + t_2 p_2 = \frac{t_1 e^{-\theta t_1}}{e^{-\theta t_1} + e^{-\theta t_2}} + \frac{t_2 e^{-\theta t_2}}{e^{-\theta t_1} + e^{-\theta t_2}}, \tag{11}$$

Assuming a small improvement in the free flow travel cost of link 1, the paradox occurs when the improvement results in a higher average travel cost. For different travel demand levels, the paradox areas of the fixed-cost case are the same; it can be easily obtained from the partial derivative of the average travel cost with respect to the improved link's cost, which has been studied by Yao and Chen (2014). For the congestion case, however, the condition under which paradox happens is related to the demand level, and the derivative is too complicated to be analytically obtained. Even in this simple two-link network, it is needed to solve a nonlinear equation contains exponential terms to obtain the equilibrium state. Therefore, an alternative numerical method is applied: for any given t_{02}, Q and θ, the boundary of paradox happens or not can be obtained by finding t_{01} which maximises the $\bar{C}$.

Figure 2 shows the differences in paradox area of the three cost functions with different demand levels when $\theta = 0.4$. The upper left part of each curve is no paradox area, and the lower right part of each curve is the paradox area, main findings are as follows:

- Congestion effect has a significant influence to the stochastic assignment paradox; the paradox areas exhibit different features when different cost functions are applied.
- It shows that the paradox area of congestion case is closer to the fixed-cost case when demand level is lower. It is intuitively understandable. Because when demand level is low, the actual travel cost is closer to the free flow travel cost, the flow distribution of congestion case and fixed-cost case are less different and consequently leads to more similar paradox areas.
- In the fixed-cost case, the slope of paradox boundary curve is 1, which means $t_{01} - t_{02}$ is a constant (Yao and Chen, 2014). However, the slopes for the congested case shown in the figure are smaller than 1, the absolute difference between free flow cost t_{01} and t_{02} enlarges with the increase of t_{02} in congested case. Despite this, the difference of actual travel cost between the two routes will be narrowed down at the equilibrium condition, because the costlier route will have less flow and have less increase in actual cost.

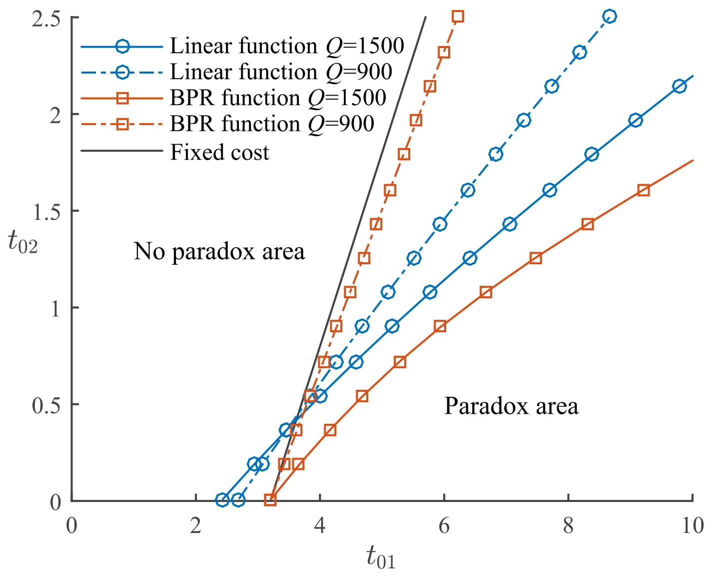


Figure 2 Paradox areas under different cost functions and demand levels when $\theta = 0.4$

3.2 Adding a link

In addition to improving an existing link, we use the network shown in Figure 3 to investigate the paradox when adding a new link. The network has the same configuration with the Braess' network, and link cost functions are set to be the same with which were used by Arnott and Small (1994), where $t_1 = 0.01x_1$, $t_2 = t_3 = 15$, $t_4 = 0.01x_4$, $t_5 = 7.5$. The link 5 is the additional link. Before link 5 is added, the demand is equally loaded on routes 1 and 2 (because the network is symmetric). After adding link 5, there are three alternative routes, namely route 1 (link 1→link 3), route 2 (link 2→link 4) and route 3 (link 1→link 5→link 4).

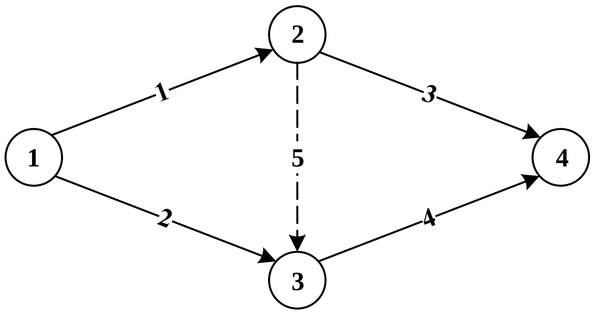
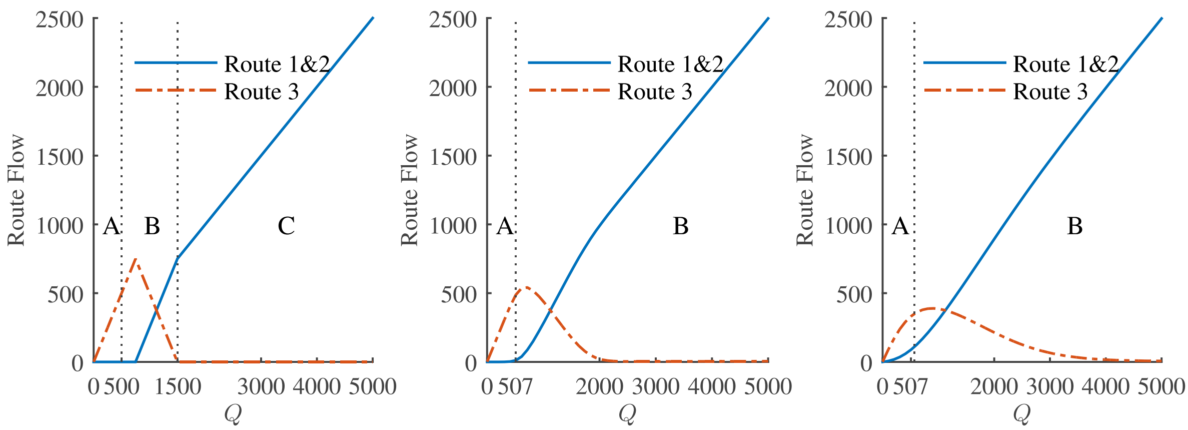


Figure 3 The Braess' network

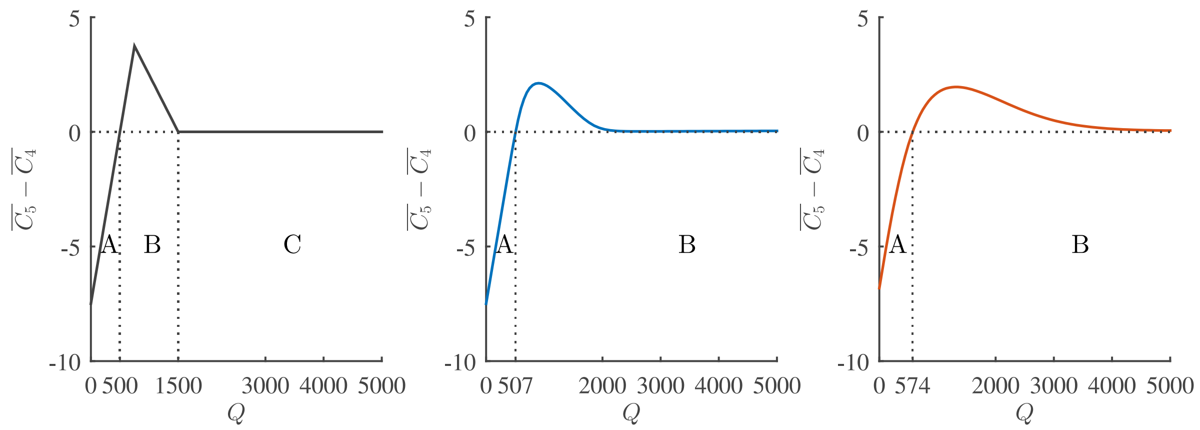
Define $\overline{C_4}$ and $\overline{C_5}$ to be the average travel cost of the network before and after link 5 is added, then $\overline{C_5} - \overline{C_4}$ can be used to indicate whether the paradox happens or not. When $\overline{C_5} - \overline{C_4} > 0$, the additional link increases the average travel cost and the paradox corresponding to the actual travel cost occurs. Otherwise, the paradox doesn't occur.

We are especially interested in how different θ in SUE influences the flow distribution as well as the demand interval in which paradox happens. The distributions of the equilibrium route flow with different traffic demands in the UE and the Logit SUE models after adding link 5 are shown in Figure 4. The value of $\overline{C_5} - \overline{C_4}$ for different traffic demands Q are also shown in Figure 5, where the part of $\overline{C_5} - \overline{C_4} > 0$ means that the paradox will happen.



(a) UE ($\theta \to +\infty$) (b) SUE ($\theta = 1.5$) (c) SUE ($\theta = 0.4$)

Figure 4 Route flows at equilibrium state with different Q values



(a) UE ($\theta \to +\infty$) (b) SUE ($\theta = 1.5$) (c) SUE ($\theta = 0.4$)

Figure 5 $\overline{C_5} - \overline{C_4}$ with different Q values

Figure 4 (a) depicts the change of equilibrium flow of UE model, it can be seen that the flow of route 1 and route 2 keeps zero at low demand level and begin to grow with the increase of travel demand after a threshold point (where the travel cost of route 3 just equals the free flow travel cost of routes 1 and 2). While the flow on route 3 will first increase and then decrease to zero after $Q = 1500$ (beyond which route 3 is always the worst alternative). For Logit SUE model, as shown in Figure 4 (b) and (c), the trend of equilibrium flow is similar to the UE model, except the flow on each route is always non-zero. It is because, theoretically, the possibility for a traveller to choose an inferior route always exists even with a very big actual travel cost. By comparison, it can be found the flow assignment result of the Logit SUE is more similar to the result of the UE for a larger θ.

It is shown in Figure 5 that the demand interval (part B) in which paradox happens of UE model has both a lower bound and an upper bound. When the demand level is relatively low (part A), the addition of link 5 decreases the average travel cost. When the demand level is higher than the upper bound (Part C), link 5 makes no difference to the average travel cost, which can be explained by Figure 4 (a) in which route 3 (link 5) has no flow in that area.

For the SUE model, however, there is no upper bound for demand level higher than which paradox does not occur. The reason is as we have mentioned, route 3 (link 5) is always possible to be chosen even its travel cost is the worst when travel demand is very big. The lowest Q for SUE paradox to happen varies with different θ, the relationship is shown in Figure 6. We can find the Q value approximates to 500 (the lower bound of UE paradox happens) when θ becomes bigger. The reason is that SUE would approximate to UE when $\theta \to +\infty$.

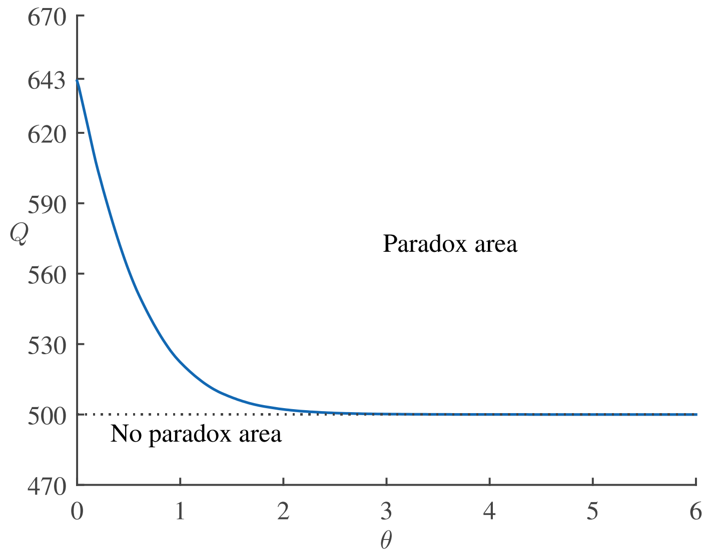


Figure 6 The lowest Q for SUE paradox measured by actual cost to happen with different θ

4. The paradox measured by perceived travel cost

Sheffi and Daganzo (1978) have concluded that the stochastic assignment paradox in a fixed-cost network will never happen if the system object is to minimise the average perceived travel cost. This conclusion, however, is not necessarily true for the congested network. We still use the Braess' network shown in Figure 3 to unfold our discussion about the perceived travel cost paradox in this section.

4.1 Improving a link

There is an important property for the average perceived travel cost. As shown in Equation (8), the partial derivative of the average perceived travel cost of an O-D pair with respect to a route's actual travel cost is the probability of choosing that route. Because the probability is always non-negative, when decreasing the actual travel cost of a route in a fixed-cost network, the average perceived travel cost of corresponding O-D pair will never increase, in other words, no paradox.

This property also holds for congested networks (that is why we can use the same Equation (9) to calculate average perceived travel cost in both cases), but the "no paradox" conclusion cannot be derived from this property in the congested case. The reason lies in the actual travel time of route varies with the flow in the congested network. After improving a route, the flow redistribution changes the actual travel cost of not only the improved route, but also other routes. Unlike the fixed-cost network, the actual travel cost of a route can never be "partially improved", and therefore the "no paradox" conclusion cannot be drawn from Equation (8).

Proposition 1: when improving an existing link in a flow-dependent network, the stochastic assignment paradox with regard to the average perceived travel cost could exist.

We use the five-link Braess' network shown in Figure 3 to show the existence of "average perceived travel cost paradox" when link 5 is improved. Fix $Q = 1000$, $\theta = 1.5$, and vary t_5 from 0 to 20 to observe the corresponding average perceived travel cost $\tilde{C}$, the results are shown in Figure 7. Firstly, as discussed by Sheffi and Daganzo (1978), it can be found that $\tilde{C} \leq \min(T_1, T_2, T_3)$ is always ture. Secondly, roughly starting from $t_5 > 4$, we can see that the average perceived travel cost decreases with the increase of t_5, which means the "paradox with regard to average perceived travel cost" exists in congested networks.

The reason of this paradox can be found by looking into the actual travel cost of each route at equilibrium state. For non-congested case, improving link 5 will decrease the actual cost of route 3, and leave the actual cost of routes 1&2 unchanged. However, in congested case, we can find from Figure 7 that when improving link 5 (decrease t_5) at around $5 < t_5 < 10$, the travel cost of routes 1&2 and even route 3 conspicuously increase. The flow-dependent and correlated feature of congested networks is the cause of the paradox.

From another aspect, the occurrence of this paradox parallels Braess' paradox. In the SUE model, no traveller can unilaterally improve his or her perceived travel cost, the state of "user's equilibrium of perceived travel cost" is not identical to the state when the system has the minimum total users' perceived travel cost, the discrepancy leads to the paradox. The same reason applies to adding a link case. For the equilibrium with total system's minimum perceived cost, the reader is referred to the Stochastic Social (or System) Optimum (SSO) (Maher et al., 2005).

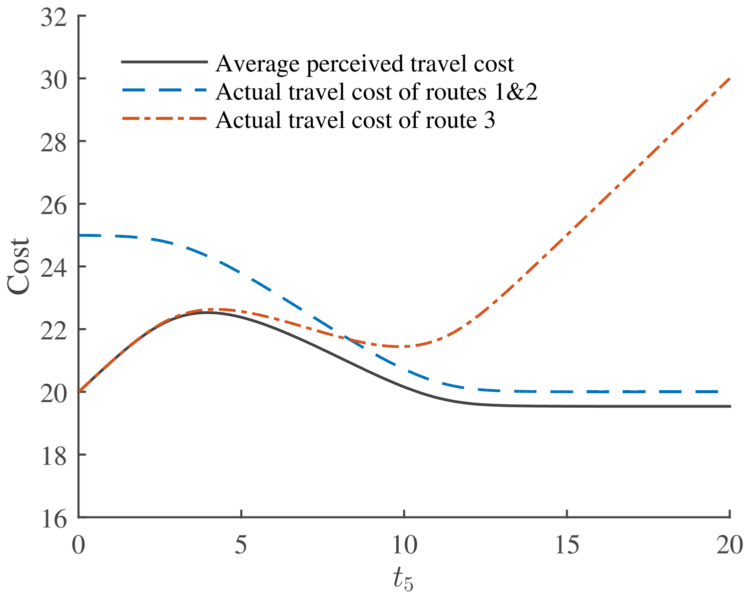


Figure 7 Change of the average perceived travel cost and the actual travel cost of different routes with different t_5, in the Braess' network when $Q = 1000$, $\theta = 1.5$

4.2 Adding a link

This section evaluates the stochastic paradox when adding a new link into the network by the measure of perceived travel cost. The average perceived travel cost of an O-D pair with N routes is as follows (The superscript numbers the O-D pair is omitted for simplicity):

$$E\left[\min_{\mathrm{i}=1}^{\mathrm{N}}\{C_i\}\right]. \tag{12}$$

Proposition 2: when adding a new link to a flow-dependent network, the stochastic assignment paradox with regard to the average perceived travel cost could exist.

To prove Proposition 2, we only need to find an example, which we will show in Figure 8 latter. But we would like to show why it is different from the non-congested case. When a new link is added, say there is the $(N+1)^{\text{th}}$ route, the average perceived cost is $E[\min_{i=1}^{N+1}\{C'_i\}]$, where C'_i is the perceived travel cost of route i after the addition. Because of the monotonicity of expectation, we have the following transformation:

$$\begin{aligned} E[\min_{i=1}^{N+1}\{C'_i\}] &= E[\min\{\min_{i=1}^{N}\{C'_i\}, C'_{N+1}\}] \\ &\leq \min\{E[\min_{i=1}^{N}\{C'_i\}], E[C'_{N+1}]\} \leq E[\min_{i=1}^{N}\{C'_i\}]. \end{aligned} \tag{13}$$

For the non-congested case, link cost (both actual and perceived) is flow-independent. Therefore the original route cost will not change after adding new link(s), thus $C_i = C'_i$, and the rightmost item of Equation (13) equals Equation (12). Therefore, the average perceived travel cost will never increase when a new link is added, which means the paradox will never happen. For the congested network, the flow redistribution resulted from the new link would change the cost of original links, and $C_i \neq C'_i$, which leads to the rightmost of Equation (13) may not equal to (be bigger than) Equation (12), the average perceived cost may increase, and the paradox could happen. Again, the cause of this paradox is the flow-dependent and correlated feature of congested network, and it is analogous to the Braess' paradox.

Similar to the last section, the Braess' network shown in Figure 3 is used to show the paradox. $\widetilde{C_4}$ and $\widetilde{C_5}$ represent average perceived travel cost before and after link 5 is added, which are calculated by Equation (9). The values of $\widetilde{C_5} - \widetilde{C_4}$ for different demand levels are shown in Figure 8, in which $\widetilde{C_5} - \widetilde{C_4} > 0$ means the paradox occurs. It is conspicuous that the paradox of stochastic assignment after adding a link in the perceived cost measure will happen, after a certain demand threshold, the paradox always exists.

Furthermore, the relationship of θ value and the lowest Q for SUE paradox to happen is shown in Figure 9. The blue dash line means the critical value of Q for the average perceived cost case. For comparison, the critical value of Q for the average travel cost case is also added in red dash-dotted line. It is clear that the critical Q values under two measures are different, the value for the average perceived cost case is higher than which in the average travel cost case when θ is small. Reversely, the critical value of Q for the average perceived cost case is smaller than which in the average travel cost case when θ is large. When $\theta \to +\infty$, both of them approach to 500, which is the result of the UE model. Specifically, the change of the critical value of Q for the average perceived cost case is non-monotonous. The curve also shows the paradox corresponding to the average perceived travel cost can happen when demand is less than 500, under which the paradox measured by average travel cost will never happen.

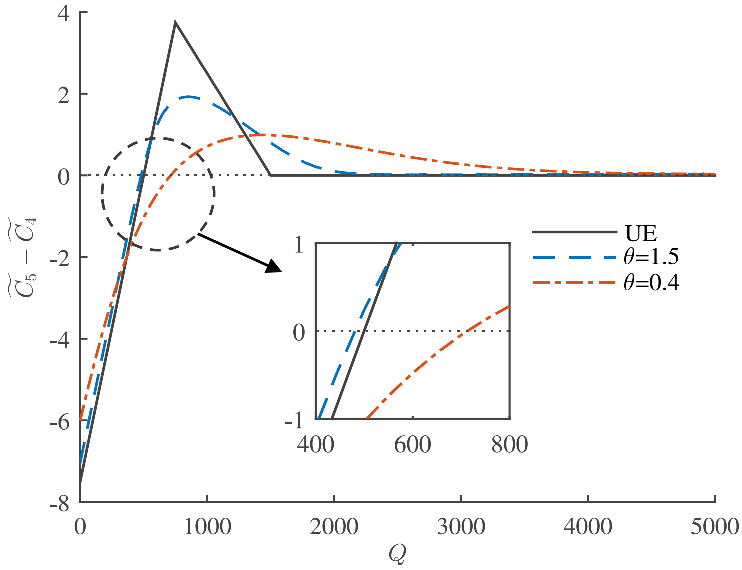


Figure 8 Difference between $\widetilde{C_5}$ and $\widetilde{C_4}$ with different Q value

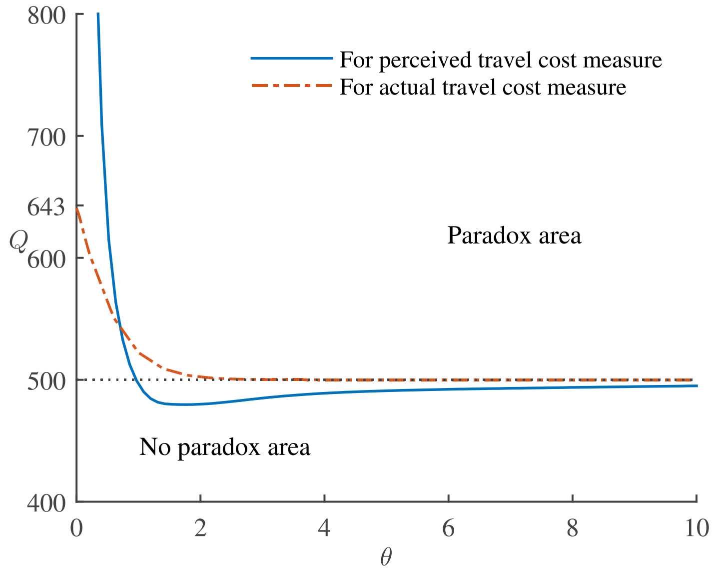


Figure 9 The lowest Q for SUE paradox to happen with different θ

5. A multiple O-D pairs' case

This section inspects how the traffic demand of an O-D pair affects the paradox of another O-D pair. We introduce link 6, link 7 and destination 5 to the Braess' network to form the two O-D pairs' network shown in Figure 10. The links' costs are inherited from the original network, they are $t_1 = 0.01x_1, t_2 = t_3 = t_6 = 15, t_4 = 0.01x_4, t_5 = 7.5, t_7 = 0.01x_7$, which lets O-D$_{1,4}$ and O-D$_{1,5}$ have the same route cost structure.

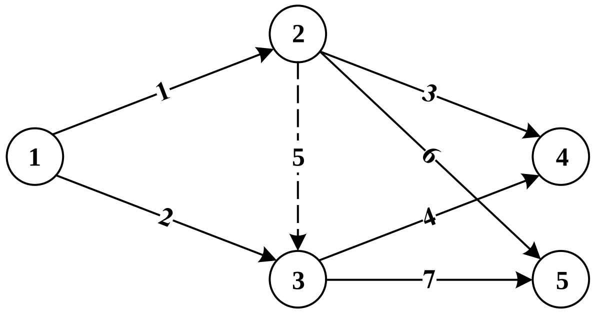


Figure 10 A two O-D pairs' network

Now we inspect whether the paradox happens after adding the link 5. When other parameters are fixed, whether the paradox occurs depends only on the traffic demand of O-D$_{1,4}$ and O-D$_{1,5}$. We are interested in how the traffic demand of O-D$_{1,5}$ affects the paradox of O-D$_{1,4}$ and the total network. Thus, the demand areas in which paradox happens are shown in figures, whose axes are the traffic demand of O-D$_{1,4}$ and O-D$_{1,5}$ respectively.

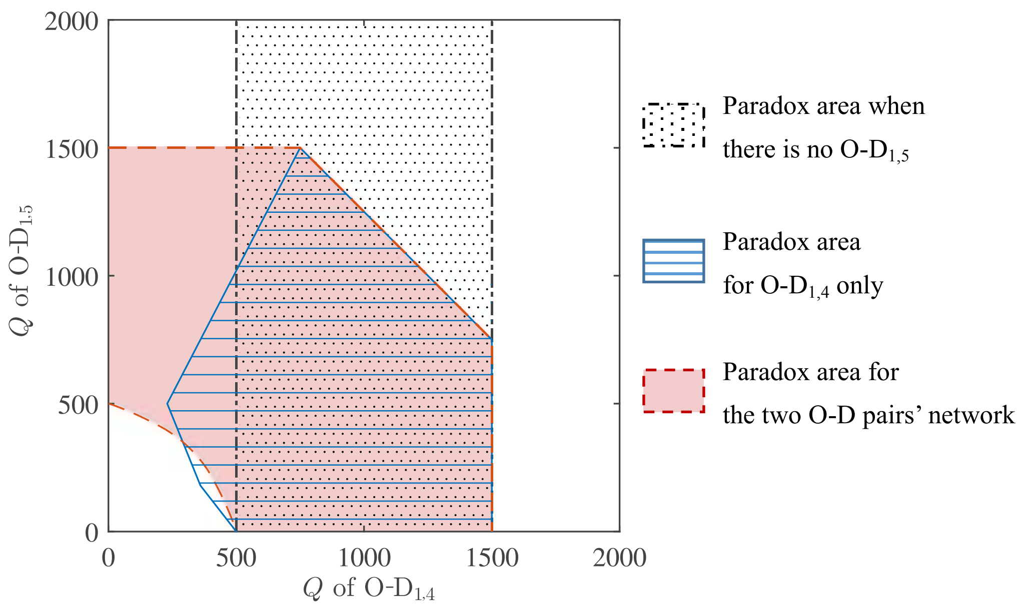
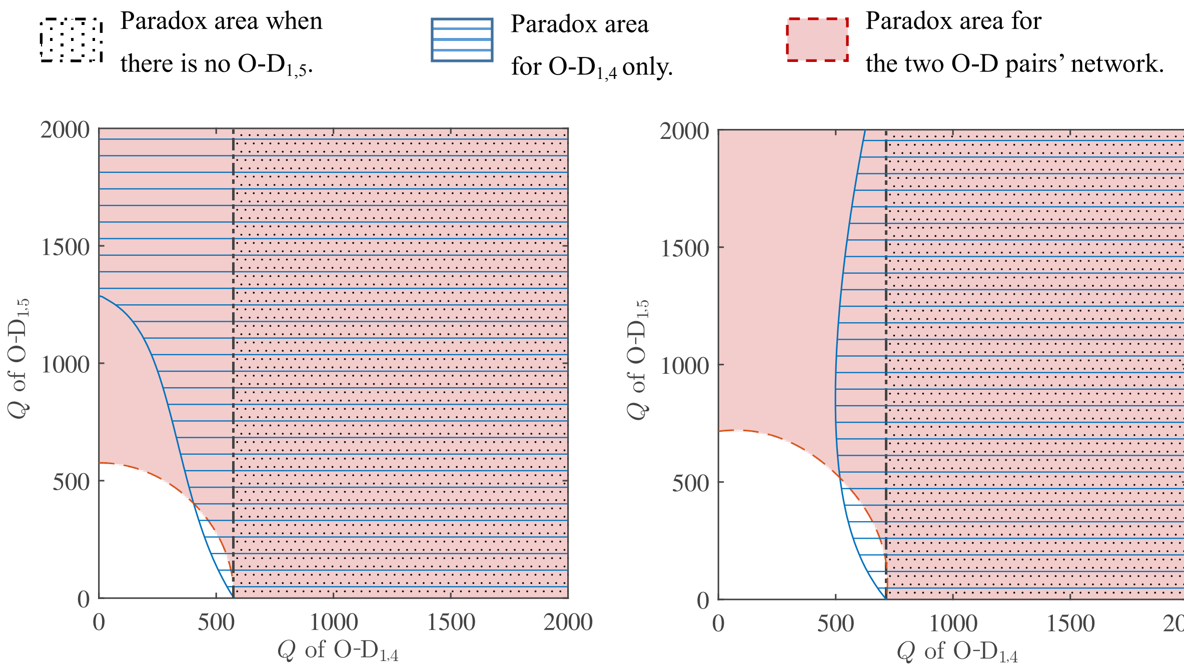


Figure 11 In user equilibrium, the demand area in which paradox happens

As is shown in Figure 11, when there is no destination 5, the paradox of O-D$_{1,4}$ happens in the demand interval (500, 1500). However, when introducing a new high demand destination 5 (e.g. when traffic demand of the new O-D$_{1,5}$ larger than 1500), the paradox in the O-D$_{1,4}$ disappears. On the contrary, some originally no-paradox areas transfer to paradox areas after the introduction of the new O-D pair when certain demand condition met. For the stochastic assignment measured by actual time and perceived time, their paradox areas are shown in Figure 12. Although in different forms, it can also be found that the demand level of O-D$_{1,5}$ has significant effect in the stochastic assignment paradox feature of O-D$_{1,4}$ and total network. Similar to section 3.1, stochastic assignment paradox of this network always exists in high demand level. It is because, theoretically, the worst route which passes link 5 is always possible to be chosen in the stochastic assignment.



(a) Measured by actual travel cost (b) Measured by perceived travel cost

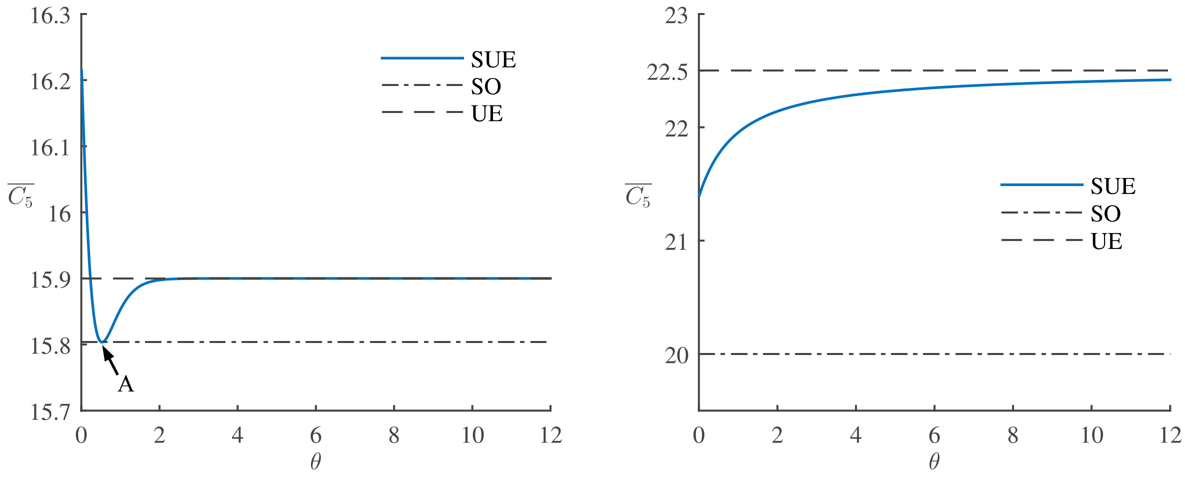
Figure 12 In stochastic user equilibrium, when $\theta = 0.4$, the demand area in which paradox happens

This section shows how the traffic paradox is affected by the interaction of traffic demands from different O-D pairs. The practical implication is twofold. On the one hand, removing negative-effect links is not the only way to reduce paradox, proper re-allocation of O-D demands also helps to alleviate the inefficiency of these links. On the other hand, we should also be aware of that the function of some links may be undermined by some unwanted changes (e.g., a newly developed shopping centre) in the regular O-D distribution of the network, even from positive to negative effect.

6. Information paradox

Intuitively, more accurate information about the travel cost of roads helps one to choose a less expensive route, and thus decrease average travel cost. However, many studies have shown that more information may have a counter effect. In the Logit SUE model, the dispersion parameter θ can be used to mark the amount of information obtained by travellers. When $\theta \to +\infty$, travellers have full information and always choose the route with least cost (equivalent to UE), when $\theta \rightarrow 0^{+}$ travellers have zero information of network and thus uniformly choose each route. Under different demands and road conditions, average travel cost of network may increase with the increase of the information level, which is referred as information paradox in this paper.

For the five-link Braess' network shown in Figure 3. Figure 13 shows the change of average travel cost $\overline{C_5}$ with regard to θ under the demand level of $Q = 420$ and $Q = 1000$. Both Figure 13 (a) and (b) show $\overline{C_5}$ could increase with a higher information level. In Figure 13 (a), the average travel cost of SUE model reaches to the system optimal solution at point A[1], after point A, $\overline{C_5}$ begins to increase and approximates to the UE with the increase of θ. In the demand level of 1000, zero information case has the minimum average travel cost in SUE model, and $\overline{C_5}$ monotonically increases with θ. For the full picture of the $\overline{C_5}$ with regard to θ and Q, we can look at the contours in Figure 14 (b), it is conspicuous that average travel cost increases with the increase of θ in certain demand level.



(a) $Q = 420$ (b) $Q = 1000$

Figure 13 Change of the average travel cost with respect to θ

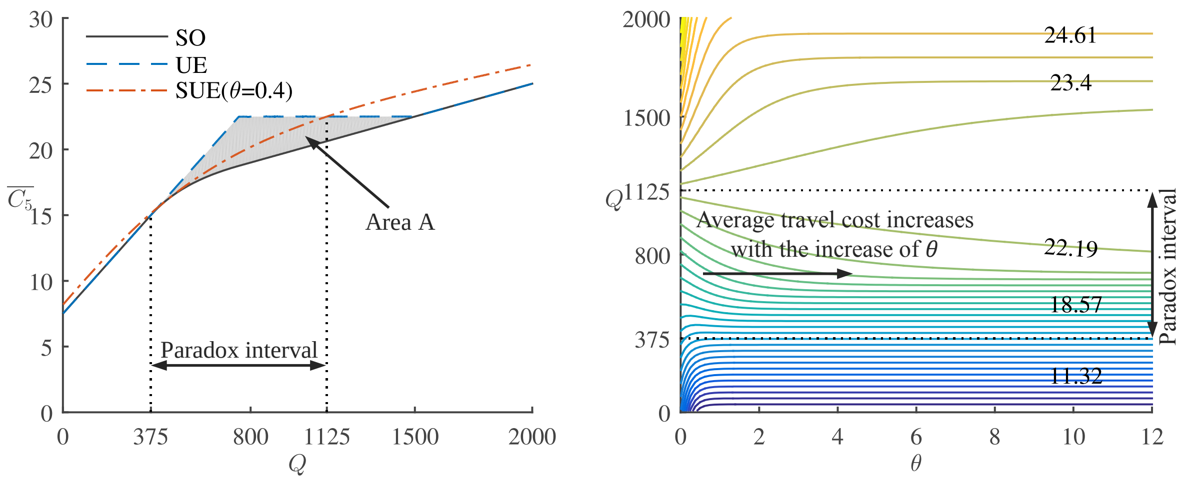
Proposition 3: if there is a certain information level $\tilde{\theta}$, under which the average travel cost of the SUE model is less than which of the UE model, the information paradox will happen.

Proposition 3 is immediate, since in the full information case, SUE→UE, if the cost under information level $\tilde{\theta}$ is less than the full information case, the information paradox therefore

[1] Because of the special structure of the cost functions in the network, route 1 and route 2 always have the same cost and therefore have the same flow. Changing θ can adjust the flow proportion between routes 1&2 and route 3 and get the same solution with SO. However, in some conditions of SO, inferior routes (routes 1&2) have more flow, these conditions can never be reached in SUE by adjusting θ, because the SUE model always assigns more flow to the superior route, such condition can be seen in Figure 13 (b).

happens.

To further understand under which demand will information paradox happen, change of average travel cost with respect to Q in different models and contours of average travel cost with respect to Q and θ are shown in Figure 14. According to the proposition 3 and as is shown in Figure 14 (a), the curve of the SUE model traverses the gray area A, which means the average travel cost of the SUE model is less than which of the UE model, and information paradox exists. To obtain the boundary of whether information paradox happens or not, we need to find a demand level at which the minimum average travel cost of the SUE model among all information level equals to the average travel cost of the UE model. The demand interval for the occurrence of information paradox has been shown in Figure 14, it is interesting to see the information paradox never occurs when UE equals SO. It is because the average travel cost of the SUE cannot be better than the system optimal solution, when UE=SO, the $\tilde{\theta}$ mentioned in proposition 3 does not exist.



(a) Change of average travel cost with respect to Q in different model

(b) Contours of average travel cost with respect to Q and θ

Figure 14 Demand interval for information paradox to happen

From the perspective of average perceived travel cost, interpreting θ as travellers' unmeasured preference rather than information level should be more justified, since perceived travel cost is related to individual's utility. The traveller's preference is basically caused by the route characteristics and the heterogeneity of routes and travellers, and a small θ implies a wider heterogeneity. Thus, the paradox in this section also implies that the reduction of heterogeneity among travellers/routes may sometimes exacerbate congestion.

7. Conclusions

This paper analyses the effects of congestion and stochastic perceived error on traffic paradox under the traditional Logit SUE model, by the measure of both actual and perceived travel cost. Besides, a two O-D pairs' network is used to show how the interaction between traffic demands of different O-D pairs affects the occurrence traffic paradox. Further, the information paradox in which a higher stochastic perceived error may decrease travellers' average travel cost is also analysed. Main conclusions are summarised as follows:

(1) Congestion effect has significant influence on the road condition under which stochastic paradox happens; different cost functions have different effects. When congestion level is less intense, the paradox condition is more similar to the fixed-cost case. On the other hand, perceived error also influences the demand interval where Braess' paradox happens, the less the perceived error is, the closer the boundary of the paradox interval is to the deterministic UE assignment.
(2) Different from the fixed-cost network, the stochastic assignment paradox corresponding to the average perceived travel cost exists in the congested network, both in the case of improving an existing link and adding/deleting a link. This parallels Braess' paradox, in the SUE model, the state of "user's equilibrium of perceived travel cost" is not identical to the state when the total system has the minimum perceived travel cost, and the discrepancy causes the paradox.
(3) The demand of a O-D pair has a significant influence to the traffic efficiency of other O-D pairs. Whether the traffic paradox of a specific O-D pair / total network occurs depends on the distribution of O-D demand among the network.
(4) The information paradox when more information counter-intuitively decreases travellers' average travel cost could happen in specific demand interval, in which the average travel cost of the SUE can be lower than which of UE by adjusting the stochastic perceived error. The condition when UE equals SO could help to avoid information paradox. From another perspective, under certain circumstance, a proper degree of heterogeneity among routes/travellers also helps to alleviate congestion.

Our research helps traffic practitioners to understand the interactional impact of congestion and stochastic perceived error, and therefore to avoid incorrect judgments or unnecessary construction in real life. For example, in section 3.2, the example shows that the paradox of the SUE model always exists in high congestion (demand) level, while this is not the case of the UE model. Think about a circumstance where several equally congested arteries are connected by some low-quality links; constructing new links like that will only aggravate congestion. The property of perceived error ensures that some travellers will always choose those links; they shift among equally congested arteries, but the travel time cannot be reduced. Section 5 demonstrates that removing negative-effect links is not the only way to reduce paradox, proper re-allocation of O-D demands also helps to alleviate the inefficiency of these links. The application scope can be further widened, such as designing one-way street is equivalent to delete a link. In addition, different congestion level and different parameters in the model have significantly different outcomes; traffic practitioner should be fully aware of this to make a reliable decision.

With the broad application of Advanced Traveller Information Systems (ATIS), travellers nowadays have more accurate information about roads' conditions. As is shown in our example, however, this not necessarily helps to improve the network performance. By adjusting the amount of information or providing information only to part of travellers under certain condition may have a better effect. A network structure or demand level in which UE is more close to SO also helps to avoid information paradox.

Further research includes: (1) analyse the paradox under the SUE condition in large networks (Youn et al., 2008; Jansuwan and Chen, 2015; Çolak et al., 2016); (2) seek better traffic network evaluation indicators, and use different evaluation indicators for different networks and different planning purposes; (3) evaluate the paradox features in other route choice models which consider congestion and stochastic perceived error (e.g., various extended logit-based models of Prashker and Bekhor (2004) and Chen et al. (2012); weibit-based models of Castillo et al.(2008) and Kitthamkesorn and Chen (2013, 2014); and hybrid logit-weibit-based models of Yao and Chen (2014) and Xu et al. (2015)); (4) study the paradox under dynamic traffic assignment (Arnott et al., 1993; Akamatsu, 2000; Nagurney and Parkes, 2007), and compare the results of theses models.

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