Traffic State Estimation from Vehicle Trajectories with Anisotropic Gaussian Processes

Fan Wu[1] Zhanhong Cheng[2] Huiyu Chen[1] Tony Z. Qiu[1] Lijun Sun[2]

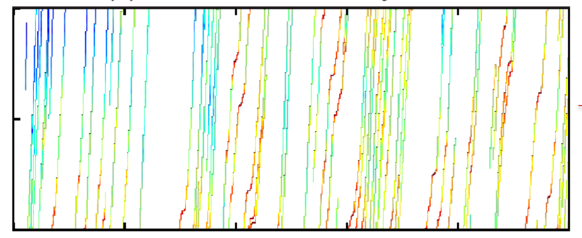
[1]Department of Civil and Environmental Engineering, University of Alberta, Edmonton, T6G 1H9, Alberta, Canada; [2]Department of Civil Engineering, McGill University, Montreal, H3A 0C3, Quebec, Canada

INTRODUCTION

What is Traffic State Estimation (TSE)?

- Traffic state estimation (TSE) refers to the inference of traffic state variables, such as density, speed, or other relevant variables, in a spatiotemporal domain by using partially observed traffic data from various detectors (e.g., loop detectors, camera, probe vehicles, connected vehicles).

(b) Observed data 5% trajectories



Given the observed data (e.g., from float vehicles, float vehicles, or detectors) to estimate the traffic speed at the blanked location.

Why TSE?

- Intelligent transportation system relies heavily on accurate traffic state information.
- The number of traffic detectors is limited.
- Only Connected Vehicles (CVs) can obtain real-time information when mixed with traditional Human-Driven Vehicles (HDVs).

Existing TSE methods

- Model-based TSE may not always be accurate because it may not fully capture the complexity of real-world traffic.
- Data-driven approaches typically require a large external training dataset and a validation dataset with full information, and they usually do not provide uncertainty quantification.

Contributions of using Gaussian Processes (GP) in TSE

- A GP model with rotated anisotropic kernels is proposed for TSE. The rotation angle can be estimated from partially observed data, which offers valuable insights into the speed of congestion propagation.
- The proposed GP-based TSE method is a purely data-driven approach that does not require an external training dataset.
- The proposed model provides statistical uncertainty quantification for the estimation.
- The multi-output GP model is proposed for TSE on multiple lanes, which leverages the correlation between the traffic states of different lanes to improve TSE accuracy.
- Extensive experiments conducted on two real-world datasets, featuring varying CV penetration rates and diverse detector types, showcase the superiority of the proposed GP-based TSE method.

PROBLEM DEFINITION

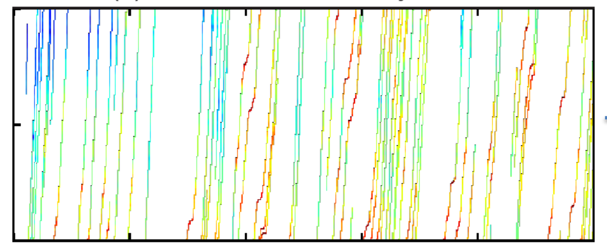
Given:

- Traffic speed $\mathbf{y}_o = \{y_i\}_{i=1}^{n}$ at a n observed locations $X_o = \{\mathbf{x}_i\}_{i=1}^{n}$
- where $\mathbf{x}_i = [s_i, t_i]^\top$ represents the spatiotemporal coordinate.

Goal:

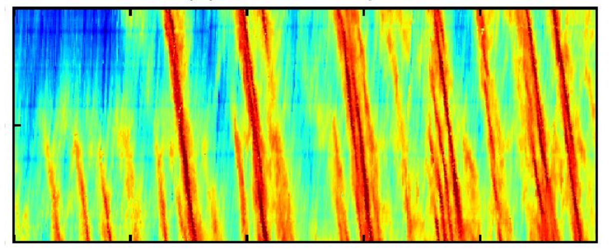
- Estimate the traffic speed distribution $p(\boldsymbol{y}_*)$ at unknown locations X_*.

(b) Observed data 5% trajectories



$$f(\mathbf{x}) \sim \mathcal{GP}(\mu, k)$$

(a) Real traffic speed

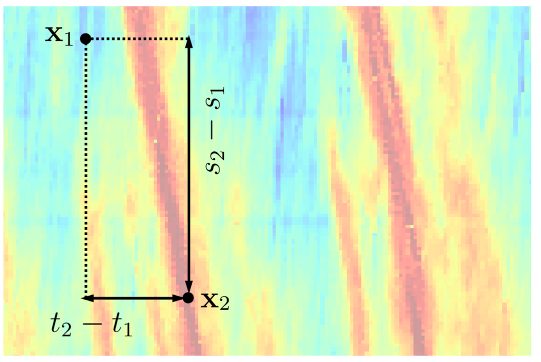


Assumptions:

- We assume the observed traffic state y_i consists of a ground truth value f_i and an i.i.d. noise term ε_i : $y_i = f_i + \varepsilon_i$
- Assume f is a function of the coordinate. Then we can impose a GP prior to the function: $f(\mathbf{x}) \sim \mathcal{GP}(\mu, k)$
- meaning any finite collection of $\mathbf{f} \in \mathbb{R}^n$ at X has a joint Gaussian distribution: $\mathbf{f} = f(X) = [f(\mathbf{x}_1), \cdots, f(\mathbf{x}_n)]^\top \sim \mathcal{N}(\mathbf{0}, K)$
- where the covariance matrix K is determined by a kernel function k, such that $K[i,j] = k(\mathbf{x}_i, \mathbf{x}_i)$. For example, a commonly used squared exponential (SE) kernel takes the form: $k_{\mathrm{SE}}(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \exp\left(-\frac{1}{2\ell^2}\|\mathbf{x}_i - \mathbf{x}_j\|^2\right)$

METHODOLOGY

Difficulties in common GP approaches:



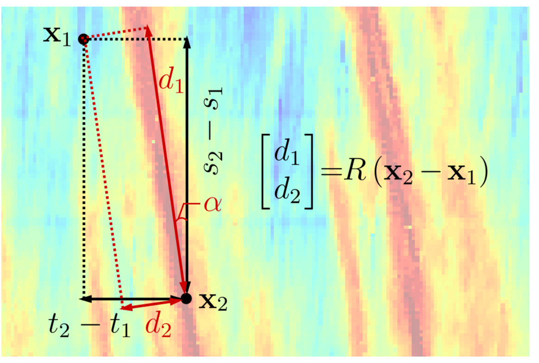
$$k(\mathbf{x}_1, \mathbf{x}_2) = k_s(s_1, s_2) k_t(t_1, t_2)$$

$$d(\mathbf{x}_1, \mathbf{x}_2)^2 = (\mathbf{x}_1 - \mathbf{x}_2)^\top \begin{bmatrix} \ell_s^{-2} & 0 \\ 0 & \ell_t^{-2} \end{bmatrix} (\mathbf{x}_1 - \mathbf{x}_2) = (\mathbf{x}_1 - \mathbf{x}_2)^\top M (\mathbf{x}_1 - \mathbf{x}_2)$$

$$k_{\mathrm{SE}}(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \exp\left(-\frac{1}{2} d(\mathbf{x}_i, \mathbf{x}_j)^2\right).$$

- Using the multiplication of a spatial kernel and a temporal kernel.
- Failed to capture directional correlation in the traffic propagation.

Rotated anisotropic kernel:



$$d_{\mathrm{rot}}(\mathbf{x}_i, \mathbf{x}_j)^2 = (R(\mathbf{x}_i - \mathbf{x}_j))^\top M (R(\mathbf{x}_i - \mathbf{x}_j)) = (\mathbf{x}_i - \mathbf{x}_j)^\top (R^\top M R)(\mathbf{x}_i - \mathbf{x}_j),$$

$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}.$$

$$k_{\mathrm{SE}}(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \exp\left(-\frac{1}{2} d_{\mathrm{rot}}(\mathbf{x}_i, \mathbf{x}_j)^2\right).$$

- Introducing a rotation matrix R.
- The rotation angle α can be learned from data.
- α provides insight into the speed of congestion propagation.

Inference with variational sparse GP:

- Estimate hyper-parameters (including kernel parameters and inducing variables) using variational sparse GP (Titsias, 2009).
- TSE using conditional Gaussian distribution $p(\mathbf{f}_* \mid X_*, X_o, \mathbf{y}_o) \sim \mathcal{N}\left(\bar{\mathbf{f}}_*, \operatorname{cov}(\mathbf{f}_*)\right)$

Multi-output GP:

- The TSE in multiple lanes can be naturally modeled using a multi-output GP model, also known as a co-regionalized GP.

EXPERIMENT & RESULTS

Datasets:

- **The NGSIM data**: trajectories extracted from video cameras on lane 2 of US highway 101, covers a longer road segment of 600 meters and a larger time range of 2500 seconds. We extract the complete data and focus on the traffic state at a 200 × 500 spatiotemporal grid with a resolution of 3 meters and 5 seconds.
- **The HighD data**: naturalistic trajectories recorded using drones on German highways track ID 25, extract traffic state in a spatiotemporal grid of size 100 × 220 with a resolution of 4 meters and 5 seconds, representing a domain of 400 meters and 1100 seconds.

Baseline models:

- Adaptive smoothing interpolation method (ASM)
- Spatiotemporal Hankel Low-Rank Tensor Completion (STH-LRTC)
- Gaussian process regression with standard ARD Matérn kernels (GP-ARD)

Results:

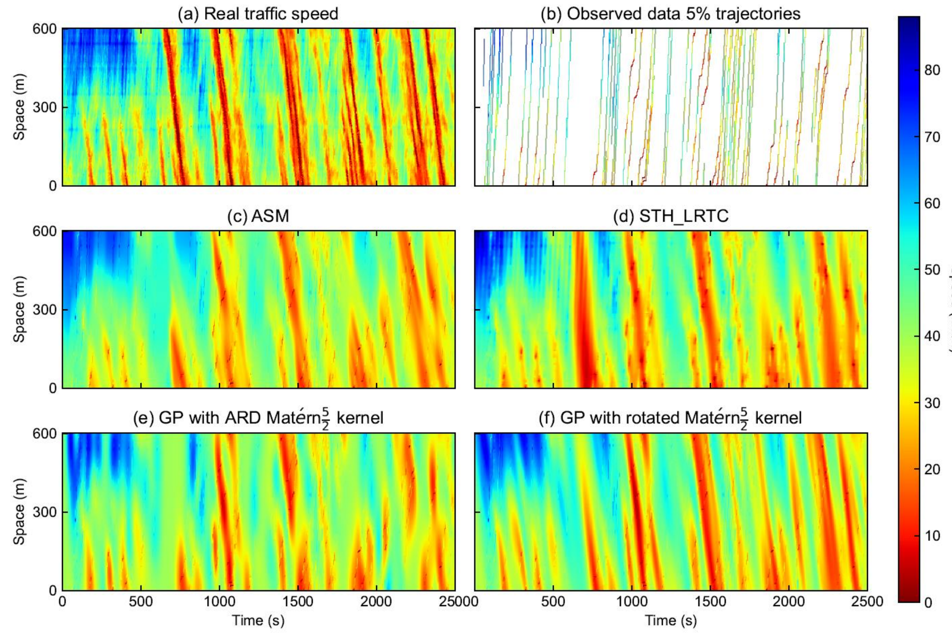
PERFORMANCE OF THE PROPOSED METHOD AND BASELINES UNDER DIFFERENT PENETRATION RATES. (NGSIM), MEAN (STD).

Method	ASM		STH-LRTC		GP-ARD		GP-rotated		P-GP-rotated	
Rate	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE
0.05	5.59 (0.36)	7.81 (0.56)	5.51 (1.36)	7.94 (2.38)	6.02 (0.36)	8.62 (0.56)	**4.85 (0.31)**	6.74 (0.56)	4.97 (0.29)	**6.74 (0.47)**
0.1	4.42 (0.17)	6.28 (0.27)	4.19 (1.39)	7.43 (5.62)	4.35 (0.30)	6.42 (0.58)	3.82 (0.22)	5.44 (0.47)	**3.79 (0.13)**	**5.19 (0.22)**
0.2	3.53 (0.10)	5.31 (0.14)	3.01 (1.26)	6.16 (7.14)	3.07 (0.14)	4.61 (0.26)	**2.81 (0.10)**	**4.10 (0.19)**	2.98 (0.08)	4.28 (0.12)
0.3	2.93 (0.06)	4.69 (0.10)	**2.09 (0.05)**	**3.17 (0.12)**	2.43 (0.06)	3.77 (0.11)	2.27 (0.05)	3.43 (0.10)	2.48 (0.05)	3.75 (0.08)
0.4	2.44 (0.06)	4.21 (0.10)	**1.75 (0.05)**	**2.81 (0.12)**	2.03 (0.06)	3.35 (0.11)	1.92 (0.05)	3.08 (0.09)	2.09 (0.05)	3.37 (0.09)
0.5	1.99 (0.06)	3.75 (0.09)	**1.43 (0.04)**	**2.46 (0.11)**	1.67 (0.04)	2.96 (0.10)	1.58 (0.04)	2.71 (0.09)	1.71 (0.04)	2.98 (0.07)

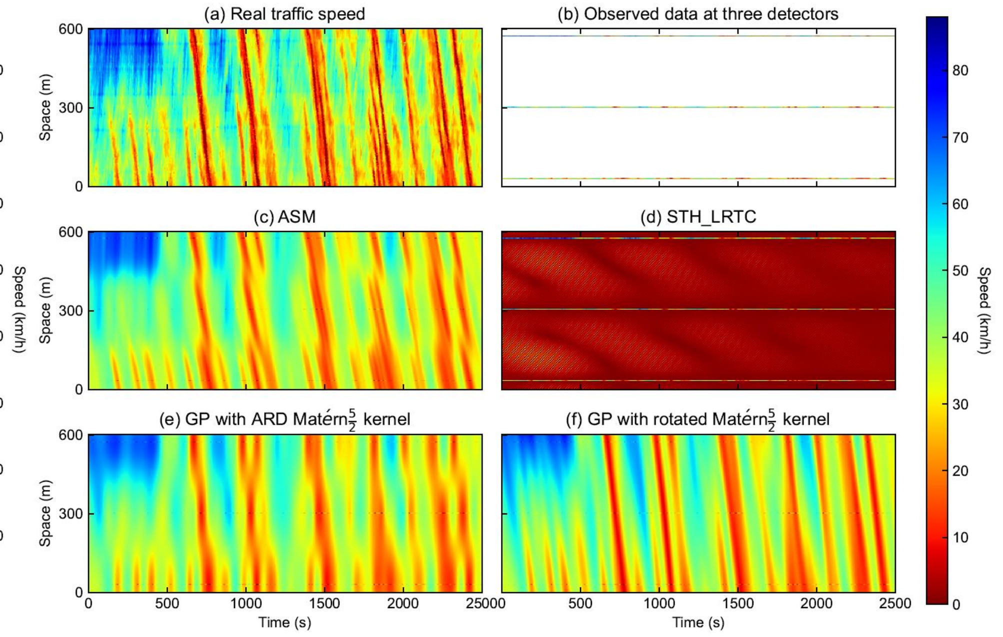
- The congestion propagation speed estimated by GP is -19.87 km/h. In comparison, a -15 km/h was adopted in ASM by Treiber et al. (2011).
- Our model is superior when the CV penetration rate is low.

EXPERIMENT & RESULTS

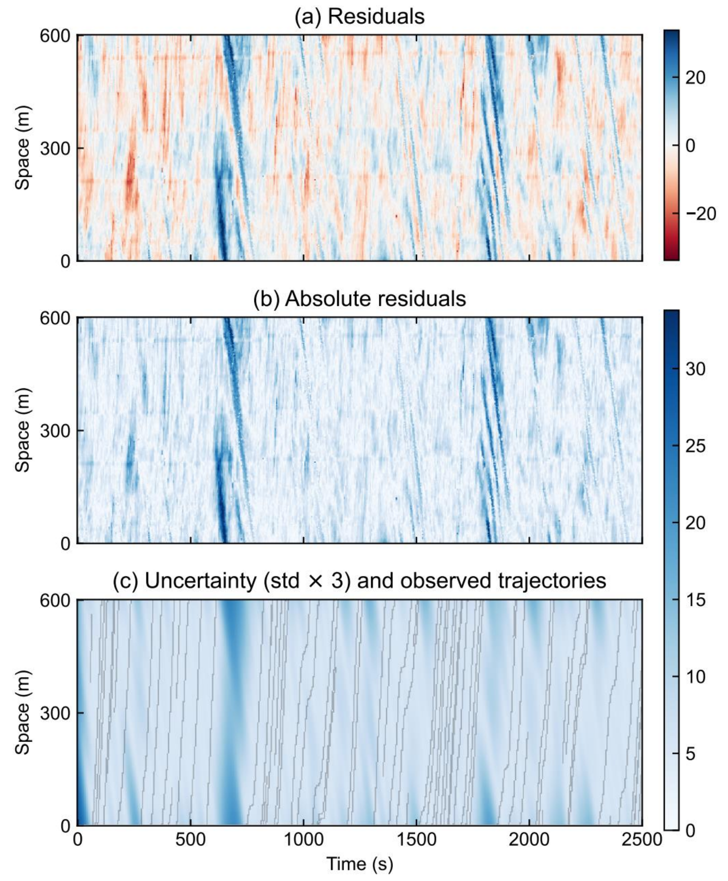
TSE experiment with 5% observed trajectories:



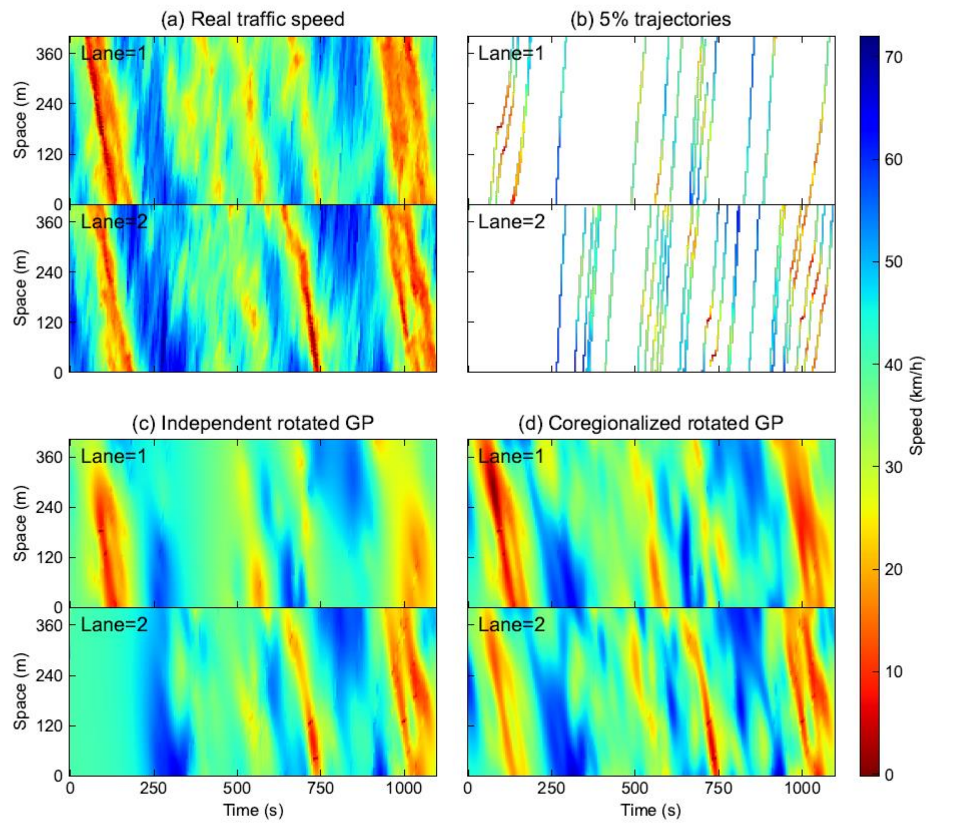
TSE from loop detectors:



Uncertainty quantification:



TSE in multiple lanes:



Computational time:

NGSIM				
Rate	ASM	STH-LRTC	GP-rotated	P-GP-rotated
0.05	7.40 (0.57)	908.21 (38.61)^a	27.30 (2.92)	**3.84 (0.27)**
0.1	14.18 (0.43)	850.90 (19.61)^a	77.54 (4.68)	**9.25 (0.42)**
0.2	26.77 (0.92)	206.72 (1.85)	153.07 (3.67)	**13.43 (1.68)**
0.3	38.38 (1.38)	199.99 (1.77)	204.61 (2.71)	**13.97 (0.19)**
0.4	48.15 (3.98)	196.09 (2.66)	245.37 (3.76)	**14.94 (0.16)**
0.5	54.21 (2.83)	191.46 (1.93)	280.01 (5.28)	**15.85 (0.26)**

HighD				
Rate	ASM	STH-LRTC	GP-rotated	P-GP-rotated
0.05	0.46 (0.03)	67.61 (3.38)	11.97 (1.89)	**0.35 (0.05)**
0.1	0.87 (0.04)	823.65 (10.74)^b	12.54 (0.29)	**0.42 (0.05)**
0.2	1.67 (0.07)	54.29 (1.39)	19.86 (0.40)	**0.83 (0.16)**
0.3	2.30 (0.05)	51.46 (1.06)	29.78 (0.66)	**1.25 (0.15)**
0.4	2.84 (0.08)	49.50 (0.89)	40.46 (0.86)	**1.63 (0.21)**
0.5	3.22 (0.09)	48.14 (0.82)	50.93 (1.49)	**2.27 (0.21)**

[a] Delay-embedding lengths $\tau_s = 50$, $\tau_t = 50$.
[b] Delay-embedding lengths $\tau_s = 60$, $\tau_t = 50$.

CONCLUSION & DISCUSSIONS

Conclusion:

- This paper presents a novel approach for traffic speed estimation using Gaussian process regression with a rotated kernel parametrization. The rotated kernel is designed to model anisotropic traffic flow, allowing for capturing the directional dependence of traffic wave propagation.
- The results on two real-world datasets show that the proposed method outperforms other state-of-the-art methods in terms of both estimation accuracy, robustness, and computational efficiency.
- The proposed method is a promising approach for traffic speed estimation and capturing directional traffic flow patterns, providing a critical solution to showcase the full traffic information with a limited CV penetration rate.